Superfluid Transition in a Rotating Fermi Gas with Resonant Interactions

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We study a rotating atomic Fermi gas near a narrow $s$-wave Feshbach resonance in a uniaxial trap with frequencies $\Omega_1$, $\Omega_2$. We predict the upper-critical angular velocity, $\omega_{c2}(\delta, T)$, as a function of temperature $T$ and detuning $\delta$ across the BEC-BCS crossover. The suppression of superfluidity at $\omega_{c2}$ is distinct in the BCS and BEC regimes, with the former controlled by depairing and the latter by the dilution of bosonic molecules. At low $T$ and $\Omega_1 \ll \Omega_2$, in the BCS and crossover regimes of $0 \leq \delta \leq \delta_c$, $\omega_{c2}$ is implicitly given by $h \left( \omega_{c2}^2 + \Omega_1^2 \right) = 2 \Delta / h \Omega_1 / \epsilon_F$, vanishing as $\omega_{c2} \sim \Omega_1 \left( 1 - \delta / \delta_c \right)^{1/2}$ near $\delta_c = 2 \epsilon_F + \frac{2}{\gamma} \epsilon_F \ln(\epsilon_F / h \Omega_1)$ (with $\Delta$ the BCS gap and $\gamma$ the resonance width), and extending the bulk result $h \omega_{c2} = 2 \Delta^2 / \epsilon_F$ to a trap. In the BEC regime of $\delta < 0$ we find $\omega_{c2} \rightarrow \Omega_1^2$, where molecular superfluidity is destroyed only by large quantum fluctuations associated with comparable boson and vortex densities.

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Recent advances in atomic gases near a Feshbach resonance (FR) have led to the realization of resonantly paired atomic superfluids [1,2]. The proximity to a FR allows a tunability of the pairing interaction, thereby permitting unprecedented access to fermionic superfluidity ranging from a weakly paired BCS regime to a strongly paired molecular BEC regime.

A fundamental aspect of a superfluid is its nonclassical response to an imposed rotation. Unable to exhibit rigid body rotation, a superfluid rotates by nucleating a vortex array with density $n_v = m \omega / \pi h$ set by the rotation rate $\omega$. Although considerable progress has been made in elucidating the properties of rotating superfluids in the BEC regime (with bosonic atoms) [3–5], considerably less is understood for a resonantly paired trapped superfluid in the BCS and crossover regimes. Recent spectacular experiments by Zwierlein et al. [6] provide strong motivation for a study of these regimes.

Additional motivation is provided by the relation of a rotating superfluid to a type-II superconductor in a magnetic field [7–9], with the Coriolis force $2m \vec{r} \times \vec{\omega}$ in the former corresponding to the Lorentz force in the latter, with the identification of $-eB/c$ with $2m \omega$. Although significant insight can be gained from this connection, it is limited to the BCS regime and does not include important ingredients that are unique to a rotating superfluid. These include the absence of screening, the centrifugal force, the trap potential, the fixed number of atoms, and the tunable resonant pairing interaction, all of which are absent in the analogous superconductor problem. In the latter a concomitant Zeeman field also appears. It can be effectively introduced into the atomic problem (but we will not do so here) by imposing a difference in the number of the two pairing atomic (hyperfine state) species [10].

In this Letter we study the effect of an imposed rotation $\omega$ on a trapped [with axially symmetric trap frequencies $\Omega_1$ and $\Omega_2$ and $\Omega = (\Omega_1 \Omega_2^2)^{1/3}$] resonantly paired superfluid near a narrow FR [i.e., the width of the resonance, $\Gamma$], is smaller than the Fermi energy $\epsilon_F = (3N)^{1/3} \hbar^2 \Omega_1^{1/3} / m$, $\gamma = \sqrt{T / \epsilon_F} < 1$, tuned through the BEC-BCS crossover [11]. Our primary prediction is the upper-critical angular velocity $\omega_{c2}(\delta, T)$ as a function of FR detuning $\delta$ and temperature $T$, illustrated for $T \rightarrow 0$ in Fig. 1 and for a range of $\delta$ as a function of $T$ in Fig. 2.

For low $T$ and $\Omega_1 \ll \Omega_2$, in the BCS and crossover regimes of $0 \leq \delta < \delta_c$, we find $\omega_{c2}$ to be implicitly given by $h \left( \omega_{c2}^2 + \Omega_1^2 \right) = 2 \Delta \omega_{c2} / h \Omega_1 / \epsilon_F^{\omega_{c2}}$, controlled by the discreteness of the rotated trap spectrum cutting off the weak Fermi-surface Cooper pairing characterized by the BCS gap $\Delta_{\omega=0} \equiv \Delta(\epsilon_F) = 8 \epsilon_F^{-2} \epsilon_F \exp(-\frac{\epsilon_F^2}{2m^2})$. In this regime, for $\frac{\gamma}{q} \ln(\epsilon_F / h \Omega_1) \gg 1$, this leads to

$$\omega_{c2}^{T=0}(\delta) = \begin{cases} \sqrt{6} \Omega_1 \sqrt{1 - \frac{\delta}{\delta_c}}, & \text{for } \delta \rightarrow \delta_c^-, \\ \Omega_1 \sqrt{1 - \left( \frac{\delta}{\delta_c} \right)^{\gamma}}, & \text{for } 0 \leq \delta \ll \delta_c, \end{cases}$$

where the behavior near $\delta_c = 2 \epsilon_F + \frac{2}{\gamma} \epsilon_F \ln(\epsilon_F / h \Omega_1)$.

FIG. 1 (color online). The upper-critical rotation rate $\omega_{c2}(\delta, T \rightarrow 0)$ as a function of the detuning $\delta$ in an isotropic trap with $10^7$ atoms, and for two FR widths $\gamma = 0, 1/3$. $\omega_{c2}$ vanishes for $\delta$ greater than $\delta_c = 2 \epsilon_F + \frac{2}{\gamma} \epsilon_F \ln(\epsilon_F / h \Omega_1)$.
corresponds to a vanishing of $T_c^{\omega=0}$ when the BCS condensation energy $\Delta^2/\epsilon_F$ drops down to the trap level spacing $h\Omega_\perp$. At low detuning $\delta \ll \delta_c$, $\omega_c^2$ rises up to but is limited below $\Omega_\perp$ by the implicit dependence entering through $\epsilon_p^\omega = \epsilon_p(1 - \omega^2/\Omega_\perp^2)^{1/3}$ and $\Delta_\omega = \Delta(\epsilon_p^\omega)$ due to the centrifugal and Coriolis forces reducing the effective trap potential and atom density. In this limit, we recover the bulk result $h\omega_c^2 = 2\Delta^2_{\omega_c}/\epsilon_{F_c}$, corresponding to Gor'kov's (fixed chemical potential) prediction for type-II superconductors [8].

In the opposite limit of $\frac{\pi}{2} \ln(\epsilon_F/h\Omega_\perp) \ll 1$, $\delta_c = 2\epsilon_F$ and $\omega_c^{T=0}$ reduces to

$$
\omega_c^{T=0}(\delta) = \omega_c(\delta) = \Omega_\perp \sqrt{1 - (\delta/2\epsilon_F)^3}, \quad \text{for } 0 < \delta \leq 2\epsilon_F.
$$

(2)

This also smoothly matches onto the $\omega_c \rightarrow \Omega_\perp$ behavior in the BEC regime of $\delta < 0$, where molecular superfluidity can only be destroyed by large quantum fluctuations associated with comparable boson and vortex densities, where it undergoes transitions to a variety of bosonic quantum Hall states [12].

As can be seen in Fig. 1 and Eq. (1), a distinction between BCS ($\delta \gg 2\epsilon_F$) and crossover ($0 < \delta \leq 2\epsilon_F$) regimes is not reflected in $\omega_c(\delta)$, controlled throughout by weak Cooper pairing. However, $T_c(\omega)$ (Fig. 2) does distinguish between these regimes, exhibiting a point of inflection at a scale $\omega_c(\delta)$ in the crossover (but not in the BCS regime). At a rotation rate $\omega < \omega_c(\delta)$ (nonzero only in the crossover regime) a finite number of molecular bosons $N_B(\omega)$ is present and $T_c(\omega)$ is set by $T_c^{\text{BEC}}[N_B(\omega, \delta)]$. At a higher rate, $\omega_c(\delta) < \omega < \omega_c(\delta)$ (as a consequence of an increased atomic density of states), $\epsilon_p^\omega$ drops below $\delta/2$ and $T_c(\omega)$ is determined by an exponentially reduced bulk BCS value $T_c^{\text{BEC}}(\epsilon_p^\omega) = \frac{\Delta(\epsilon_p^\omega)}{\pi}$ ($c = 0.577$ is the Euler-Mascheroni constant).

In the BCS regime ($\delta > 2\epsilon_F$), at slow rotation $\omega \ll \omega_c^{T=0}$, we find

$$
T_c(\omega) = T_c^{\text{BEC}}(1 - \omega^2/\Omega_\perp^2)^{1/3}, \quad \text{for } \delta \leq 0.
$$

(4)

We now sketch the derivation of these results, delaying details to a future publication [13].

A gas composed of two hyperfine species (labeled by $\sigma = \uparrow, \downarrow$) of fermionic atoms interacting through a tunable (via a “bare” detuning $\delta_0$) s-wave FR, corresponding to a closed-channel diatomic molecular state, is well characterized by a Hamiltonian $H = H_F + H_B + H_g$ [11,14,15], where

$$
H_F = \sum_\sigma \int d^3 r \psi_\sigma^\dagger(r) \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(r) - \mu - \omega L^z \right] \psi_\sigma(r),
$$

$$
H_B = \int d^3 r \phi^\dagger(r) \left[ -\frac{\hbar^2 \nabla^2}{4m} + 2V(r) - 2\mu + \delta_0 - \omega L^z \right] \phi(r),
$$

$$
H_g = g \int d^3 r [\phi^\dagger(r) \psi_\uparrow(r) \psi_\downarrow(r) + \psi_\uparrow^\dagger(r) \psi_\downarrow^\dagger(r) \phi(r)],
$$

(5)

written in the frame rotating with $\omega\xi$. Here $\psi_\sigma^\dagger(r)$ is an atomic creation operator in the hyperfine state $|\sigma\rangle$, whereas $\phi^\dagger(r)$ is the molecular creation operator. Above, $m$ is the fermion mass, $\mu$ the chemical potential, $g$ the atom-molecule interconversion amplitude, $L^z = -i\hbar \xi \cdot (r \times \nabla_r)$ the angular momentum operator, and $V(r) = \frac{1}{2} m (\Omega_\perp^2 x^2 + \Omega_\perp^2 y^2)$ the atomic trap potential.

To determine $\omega_c(\delta, T)$ we look for an instability of the normal state to a superfluid state. To this end we consider the system’s partition function $Z = \text{Tr}[e^{-\beta \mathcal{H}}] \left( \beta = 1/k_B T \right)$ and integrate out Fermi fields perturbatively in the FR coupling $g$, valid when the corresponding dimensionless coupling $\gamma = \frac{1}{4\pi g^2 (\Omega_\perp)^2}/(\hbar m)^{1/2} \ll 1$. We obtain

$$
S_\phi = -\hbar \ln[Z_0^\phi] + \int_0^{\beta \hbar} d\tau \int d^3 \mathbf{r} \phi^\dagger(\mathbf{r}, \tau) (h \partial_\tau + \hat{h}_B) \phi(\mathbf{r}, \tau)
$$

$$
\quad + \frac{1}{\hbar} \int_0^{\beta \hbar} d\tau d\tau' \int d^3 \mathbf{r} d^3 \mathbf{r}' \phi^\dagger(\mathbf{r}, \tau) \Sigma(\mathbf{r}, \mathbf{r}', \tau - \tau') \phi(\mathbf{r}', \tau'),
$$

(6)

where $\Sigma(\mathbf{r}, \mathbf{r}', \tau) = -g^2 G_F^{(0)}(\mathbf{r}, \mathbf{r}', \tau) G_F^{(0)}(\mathbf{r}, \mathbf{r}', \tau)$ is the molecular (Cooper pair) self-energy arising from molecule fluctuations into a pair of atoms governed by the free fermion propagator $G_F^{(0)}(\mathbf{r}, \mathbf{r}', \tau) = -\langle \psi_\sigma(\mathbf{r}, \tau) \psi_\sigma(\mathbf{r}', 0) \rangle_0$.
and $Z_F^{(0)} = \text{Tr}[e^{-\beta H_F}]$ is the partition function of free fermions. Higher-order corrections in $g$ lead to the inclusion of molecular interactions induced by the atomic FR, small for $\gamma \ll 1$.

The instability to the superfluid state at $\omega_c(\delta, T)$ is determined by the vanishing of the lowest eigenvalue of the inverse of the effective molecular propagator 

$$G_B^{(0)-(1)}(\mathbf{r}, \mathbf{r}'; \omega)\big|_{\omega=0} = -\langle \phi(\mathbf{r}, \mathbf{r}') | 0 \rangle$$

is the free molecular propagator and $\omega = 2\ell \pi / (\beta h)$ ($\ell \in \mathbb{Z}$) is the bosonic Matsubara frequency [16]. To lowest order in $g$, $G_B^{(0)}(\mathbf{r}, \mathbf{r}'; 0)$ is diagonalized by eigenstates of $G_B^{(0)-(1)}(\mathbf{r}, \mathbf{r}'; 0)$ [valid for a narrow resonance], which are simply rotated trap eigenstates $\phi_n(\mathbf{r})$ defined by $[-i\hbar^2 \nabla^2 / 4m + 2V(r) - \omega_c L_x \phi_n(\mathbf{r}) = \epsilon_n^{(0)} \phi_n(\mathbf{r})$, with eigenvalues $\epsilon_n = \{n_0, n, n_1\}$] [17].

$$e_n^{(0)} = \hbar \Omega_\perp (2n + 1) - 2 \hbar \omega_{\perp} n_z + \hbar \Omega \omega_{\perp} (n_0 + 1/2),$$

where $n_0 \in \mathbb{N}$, $n \in \frac{1}{2} \mathbb{N}$, and $n_1 = \{n, -n + 1, \ldots, n\}$ are the axial, radial, and angular momentum quantum numbers. The single-particle fermionic and bosonic spectra are then given by $e_n^{(0)} = e_n + \mu$ and $e_n^{(B)} = e_n^{(0)} - 2\mu$, respectively. In the weakly interacting limit the condensation is therefore into $\mathbf{n} = \{0 \equiv (0, 0, 0)\}$, a nonrotating bosonic eigenmode localized at the center of the trap [17].

Projecting the self-energy operator $\Sigma(\mathbf{r}, \mathbf{r}'; 0)$ onto the condensate $\phi_0(\mathbf{r})$ gives $\omega_c$ as the solution of the Thouless criterion $e_n^{(B)} = e_n^{(0)} - \int d^3r d^3r' \phi_0^*(\mathbf{r}) \Sigma(\mathbf{r}, \mathbf{r}'; 0) \phi_0(\mathbf{r}')$, explicitly given by

$$1 = \frac{g^2}{\delta - 2\mu} \left[ \sum_{n,n'} |R_{nn'}|^2 \frac{\tanh(\beta \epsilon_n^{(F)}/2) + \tanh(\beta \epsilon_n^{(B)}/2)}{2(\epsilon_n^{(F)} + \epsilon_n^{(B)}/2)} - \int \frac{d^3k}{(2\pi)^3} \frac{m}{\hbar^2} \right],$$

where we have expressed the bare detuning, $\delta_0$, appearing in the Hamiltonian in terms of the physical detuning $\delta = \delta_0 - g^2 \int \frac{d^3k}{(2\pi)^3} \frac{m}{\hbar^2}$ arising in the two-body s-wave scattering measurement through the scattering length $a_s = -\frac{m g^2}{4\pi \hbar^2}$. The matrix element $R_{nn'} = 2^{-3/4} \int d^3r \phi_0(\mathbf{r}) \phi_n^*(\mathbf{r}/\sqrt{2}) \phi_n^*(\mathbf{r}/\sqrt{2})$ is given by

$$|R_{nn'}| = \frac{1}{2} \sqrt{\frac{m \Omega}{2\hbar \pi n}} \frac{1}{2(\epsilon_{n}^{(F)} + \epsilon_{n'}^{(B)}/2)} \frac{\left(1 - (n_0^{(F)} + n_0^{(B)})/2\right)^{n_0^{(F)} + n_0^{(B)}/2} n_0^{(F)} n_0^{(B)}}{\left(n_0^{(F)} + n_0^{(B)}/2\right)^{n_0^{(F)} + n_0^{(B)}/2} n_0^{(F)} n_0^{(B)}},$$

$$\times \frac{1}{\sqrt{(n - n_z)! (n + n_z)! (n' - n_z')! (n' + n_z')!}},$$

Equation (8) needs to be solved together with the total atom number equation $N = \beta^{-1} d \ln Z / d \mu = \int d^3r (2\phi^2(\mathbf{r}) + \langle \psi_0(\mathbf{r}) \psi_0(\mathbf{r}) \rangle)$, fixing $\mu$ in terms of $N$, which to this order in $g$ is given by $N = N_0 + 2N_B + \mathcal{O}(g^2)$ with $N_0 = \sum_n 1 / (e^{\beta \epsilon_n^{(B)}} + 1)$ and $N_B = \sum_n 1 / (e^{\beta \epsilon_n^{(F)}} + 1)$. Analytic analysis of Eq. (8) is possible due to considerable simplifications in the limits of a narrow resonance ($\gamma \ll 1$) and a Fermi energy that is large compared to the rotating trap level spacing, $\epsilon_F \gg h(\Omega_\perp + \omega)$. The latter ensures that the fermionic trap level sums are dominated by large quantum numbers $\mathbf{n}$, $\mathbf{n}'$, allowing the Gaussian approximation $(\frac{1}{2\pi})^{n_0\epsilon_n^{(B)}} / (n_0! n_1!) = (\pi n)^{-1/2} \times \exp[-(n - n')^2 / 4n]$. Physically this approximation corresponds to a weak Coriolis force, with the atomic trajectories nearly straight lines (locally well characterized by plane waves) turning by a small fraction of the particle spacing, $n^{-1/3}$. This reduces the Thouless criterion, Eq. (8), to

$$\frac{\delta - 2\mu}{\gamma \epsilon_F \mu} = F(\beta \mu) + G \left[ \beta \mu, \beta \hbar \Omega_\perp \left(1 + \frac{\omega_c^2}{\Omega_\perp^2} \right), \beta \hbar \Omega_\perp \right],$$

where

$$G(x, a, b) = \int_0^\infty \int_0^\infty dy dp \int_0^1 dz \frac{e^{-p^2} \sqrt{y/(p\pi)} \tanh(y/2) \frac{1}{2\sqrt{\pi}} \frac{1}{\cosh(\sqrt{2}y/\pi)} \frac{1}{\sinh(\sqrt{2}y/\pi)} \frac{1}{\sqrt{\alpha + (b - a)\alpha}}}{\sqrt{1 + \frac{\cosh(\sqrt{2}y/\pi)}{\sinh(\sqrt{2}y/\pi)}}},$$

$$F(x) = \int_0^{\infty} dy \sqrt{y/(4\pi)} \left[ \frac{1}{y} \tan\left(\frac{\sqrt{y}}{2}\right) \frac{1}{\pi} \right] - \frac{1}{2},$$

$$F(x \gg 1) = \ln(8\pi / \alpha - 2 + c),$$

with $Q = p_1 x / (a + (b - a)\alpha^2)/(2\pi)$, $c_1 = \ln 4n + c$, and $\psi_0(y)$ the digamma function. Within the same approximation, the number equation reduces to

$$N = 2 \left( \frac{k_B T}{\hbar \Omega_\perp} \right)^{3/2} \left[ - \text{Li}_3(-e^{-\beta \epsilon_n^{(B)}}) + \text{Li}_3(e^{-\beta \epsilon_n^{(F)}}) \right],$$

where $\text{Li}_3(x) = \sum_{k=1}^{\infty} x^k / k^3$ is the trilogarithm function, with asymptotic forms $\text{Li}_3(-e^{-z}) = -z^3/6$ for $z \gg 1$, $\text{Li}_3(-e^{-z}) = -e^{-z}$ for $z \ll -1$, and $\text{Li}_3(1) = \zeta(3) = 1.202$. The first and second terms are the number of atoms and thermally excited molecules (with the condensate vanishing at $\omega_c$, respectively), and $\Omega_\perp(\omega) = \Omega_\perp(1 - \omega^2 / \Omega_\perp)^{1/3}$ is the effective trap frequency, reduced by the centrifugal “potential.”

In the thermodynamic limit $\beta \hbar \Omega_\perp \ll 1$ and $\omega = 0$, $G(x, 0, 0) = 0$ and (for a narrow FR, $\gamma \ll 1$) we recover the BCS-BEC crossover with $T_c(\delta)$ ranging from an ex-
ponentially small BCS-regime ($\delta > 2\epsilon_F \gg k_B T$) value
\[ k_B T_{c,\text{bulk}}^{\text{BCS}} = \frac{\sqrt{\pi}}{2} e^{2\pi^2 \epsilon_F} \exp[-2\pi^2 \epsilon_F] \] through \[ T_{c}^{\text{cross}} = T_{c}^{\text{BEC}} \left[1 - \left(\frac{\delta}{2\epsilon_F}\right)^{3/4}\right] \] in the crossover $0 < \delta < 2\epsilon_F$ regime
and saturating at \[ T_{c}^{\text{BEC}} = \left(\frac{1}{N_F}\right)^{1/3} \hbar \Omega_1^{-1/3} \] for large negative $\delta$.

At finite $\omega$ and low $T$, $\beta \Omega_\perp \gg 1$ (specializing for simplicity to an anisotropic trap $\beta \Omega_\perp \gg 1$, $\beta \Omega_z \ll 1$), using $G(x \gg 1, a \gg 1, b \ll 1) = 1 - \frac{1}{2} \ln x$,
we find an implicit equation
\[ h\omega_c^2 + \Omega^2_\perp = \frac{1}{2} e^{1+2/\Delta} \frac{\hbar}{\epsilon_F} \Omega_\perp \] with inequalities $\omega_c^2$ dependence enters through $\mu$ as the solution of the number equation, Eq. (14), that gives $\mu = \epsilon_F^\ast = (3N)^{1/3} \hbar \Omega_\perp$.

The resulting solution for $\omega_c^2$ is illustrated in Fig. 1, with asymptotics, controlled by a dimensionless parameter $\frac{\Delta}{\epsilon_F} \ln (\epsilon_F/\hbar \Omega_\perp)$, summarized in Eqs. (1) and (2).

We find that $\omega_c^2$ is driven to zero for $\delta > \delta_c = 2\epsilon_F + \frac{1}{2} \ln (\epsilon_F/\hbar \Omega_\perp)$.
This corresponds to a vanishing of $T_c^{\text{cross}}$ when the BCS condensation energy $\Delta^2/\epsilon_F$ becomes comparable to trap level spacing $\hbar \Omega_\perp$. Equivalently, this condition corresponds to an oscillator length $a_\perp = \frac{\hbar}{m \Omega_\perp}$ dropping to the coherence length $\xi = \hbar v_c / \pi \Delta$.

In the opposite limit of low diluting, $\omega_c^2(\delta \ll \delta_c) \to \Omega_\perp$, where the centrifugal and trapping potentials nearly cancel, the system becomes translationally invariant and we obtain the bulk result $\omega_c^2 = 8^{-1} h \epsilon_F^2 \Delta^2 / \epsilon_F$, corresponding to Gor’kov’s (fixed chemical potential) prediction for type-II superconductors [8]. This matches onto the BCS regime $\omega_c^2(\delta < 0) = \Omega_\perp$, where a purely molecular superfluid can only be destroyed by a sufficient dilution down to comparable boson and vortex densities, driving a quantum transition into a variety of bosonic quantum Hall states [12].

At high $T$ (near $T_0^\ast$) and slow rotation ($\omega \ll \omega_c^{T_0^\ast}$) in the BCS regime we expand $G(x \gg 1, a \gg 1, 0)$ in the limit
\[ ax = \beta^2 \mu \Omega_\perp (1 + \alpha^2/\Delta^2) \ll 1 \] (remaining in the degenerate limit, $\beta \mu = x \gg 1$), obtaining a quadratic suppression of $T_c$ with rotation and trap frequencies given in Eq. (3).
In the homogeneous limit $\omega \to \Omega_\perp$, this reduces to $T_c(\omega)/T_{c,\text{bulk}}^{\text{BCS}} = 1 - \frac{2(3)^{3/2} h |\omega| \mu / (k_B T_{c,\text{bulk}}^{\text{BCS}})^2}{(1/3)(\delta / 2\epsilon_F)^{3/2} N_F(\delta / 2\epsilon_F)^{3/2} N}$, giving a linear reduction with $|\omega|$ as expected from Abrikosov’s theory of $H_\perp(\omega^2)$ [7].

In the crossover regime at slow rotation, $\mu = \delta^2 / 2 < \epsilon_F^\ast$ and a finite fraction of the atomic Fermi sea is bound into a molecular superfluid, giving
\[ N_F(\omega, \delta) = \frac{\Delta^2}{2\pi^2 \epsilon_F^2} N \left[1 - \frac{1}{(\omega / \Omega_\perp)^2}\right] \] above the degeneracy of Landau levels in the $\omega \to \Omega_\perp$ limit, at sufficiently large $\omega$, $\epsilon_F^\ast$ drops below $\delta / 2$. The crossover frequency $\omega_c (\delta)$, defined by $\epsilon_F^\ast = \delta / 2$, corresponds (for a narrow resonance $\gamma \ll 1$) to a vanishing molecular condensate, $N_F(\omega_c, \delta) = N$, and is given by Eq. (2).

Naively, this is expected to be accurate only for $\gamma \epsilon_F / \hbar \Omega_\perp \ll 1$. However, in the opposite (thermodynamic) limit of a shallow trap, the eigenfunctions of $\Sigma(r, r'; \omega)$ and $\hbar B$ (Landau level orbitals) become identical, guaranteeing that such projection in fact gives the exact eigenvalue of $G_B^{-1}$. Consequently, we expect this approximation to be accurate as long as a far less restrictive condition, $\gamma \ll 1$, is satisfied.