Quasiparticles in the 111 state and its compressible ancestors

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We investigate the relationship of the spontaneously interlayer coherent "111" state of quantum Hall bilayers at total filling factor $\nu=1$ to "mutual" composite fermions, in which vortices in one layer are bound to electrons in the other. Pairing of the mutual composite fermions leads to the low-energy properties of the 111 state, as we explicitly demonstrate using field-theoretic techniques. Interpreting this relationship as a *mechanism* for interlayer coherence leads naturally to two candidate states with nonquantized Hall conductance: the mutual composite Fermi liquid and an interlayer coherent charge e Wigner crystal. The experimental behavior of the interlayer tunneling conductance and resistivity tensors are discussed for these states.

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Double-layer quantum Hall systems exhibit a wealth of fascinating behavior, among the most beautiful of which is the integer quantum Hall effect at *total* filling fraction $\nu=1$.[1,2] In the absence of interlayer tunneling (which can be tuned to be arbitrarily small experimentally), the mere existence of the quantum Hall effect at this filling factor is possible only due to strong Coulomb interactions, which stabilize the so-called 111 state (see below). As discussed theoretically by Girvin *et al.*, this state may also be viewed as an easy-plane pseudospin quantum Hall ferromagnet (QHFM), having spontaneously broken the U(1) symmetry corresponding to the conservation of the charge difference between the two electron gases. A remarkable richness of behavior was predicted to arise in response to in-plane magnetic fields and changes in temperature, much of which has indeed been verified experimentally.[3,4]

The 111 state occurs for small interlayer separation $d<d_c$, in which the Coulomb interactions between electrons in opposite layers are strongest. Much less well understood is the behavior of bilayers at $\nu=1$ for larger separations. In the limit $d\rightarrow\infty$, the layers become decoupled, and it is believed that each layer forms an independent compressible composite Fermi liquids[5] (ICFL's). For intermediate $d>d_c$, the ground state is not known. Moreover, in this range such bilayers exhibit unexplained and somewhat puzzling behavior. Coulomb drag measurements show a surprising low-temperature saturation of the trans-conductivity, quite different from the predictions for the ICFL state. Recent experimental measurements of the nonlinear interlayer tunneling conductance show considerable structure for $d>d_c$.[3] Surprisingly, most of this structure is preserved for $d<d_c$, being modified only in a narrow range of low voltage bias.

In this paper, we exploit the equivalence of the 111 state to a *p*-wave superconductor (pSC) of mutually composite fermions (MCF's), postulated earlier by Morinari.[6] This equivalence was also explored recently in Ref. 7, while duality techniques used in Ref. 8 are related to our later manipulations. The MCF's themselves are similar to but distinct from the usual composite fermions, and in particular in and of themselves already embody strong interlayer correlations. We explore the MCF formulation in more detail than prior treatments, demonstrating that the Lagrangian of the pSC state is *dual* to the earlier FM picture. The 111 state therefore provides an explicit realization of the (2+1)-dimensional bosonization and duality formulation espoused in Ref. 10. Our results deepen the understanding of the charged and neutral sectors of the 111 state, and their coupling to (pseudo)spin. Finally, this analysis provides two natural candidate ground states for $d\gtrsim d_c$: the mutual composite Fermi liquid (MCFL) of unpaired MCF's, and a charge e Wigner crystal with coincident pseudospin superfluidity. The latter state is one of several phases suggested in Refs. 7,8. As an experimental means of searching for these two possible intermediate states, we investigate the corresponding resistivity tensors and the interlayer tunneling conductances, finding the former in agreement with Refs. 7,8 where comparison is possible. The MCF liquid has *metallic* intralayer longitudinal resistivity and a constant finite *Hall drag* at low temperatures, but a pseudogap in the interlayer tunneling conductance. The charge e Wigner crystal is insulating, but should exhibit a sharp interlayer tunneling conductance peak at low temperatures due to interlayer phase coherence.

Simple algebraic manipulations of lowest Landau level wave functions suggest a relation between the 111 state and MCF's. Similar considerations have been successfully used to relate the 331 state to a pSC phase of ordinary composite fermions.[9] The simplest description of the QHFM is in terms of the 111,

$$\Psi_{111}=\prod_{i<j}(z_i-z_j)(w_i-w_j)\prod_{ij}(z_i-w_j)\Psi_G, \tag{1}$$

where $\Psi_G=\exp[-\Sigma_i(|z_i|^2+|w_i|^2)/4l^2]$, and l is the magnetic length, and $z,w=x+iy$. Using the useful identity $\Pi_{i<j}[(z_i-z_j)(w_i-w_j)]/\Pi_{ij}(z_i-w_j)=\det[1/(z_i-w_j)]$, this can be rewritten as

$$\Psi_{111}=\prod_{ij}(z_i-w_j)^2\det[1/(z_i-w_j)]\Psi_G. \tag{2}$$

The latter rewriting demonstrates that the 111 wave function is the product of a BCS pair wave function (the $\det[1/(z_i-w_j)]$ factor) and a phase-carrying factor $\Pi_{ij}(z_i-w_j)^2$. This phase factor can be interpreted in the usual quantum Hall fashion in terms of flux attachment. In particular, this factor is equivalent to attaching two flux quanta (or more precisely vortices) of layer-1 flux to the electrons in the second layer, and vice versa. Since an even number of flux quanta are attached to each particle, the composite objects so formed remain fermionic.

We next turn to a field-theoretic formulation of this flux attachment. Denoting the microscopic electron (annihilation) fields $c_\alpha(\mathbf{x})$ [where $\alpha=\uparrow,\downarrow$ indexes the two pseudospin components (layers)], we define MCF operators $\psi_\alpha(\mathbf{x}) = \exp[iK_{\alpha\beta}\int d^2\mathbf{x}'\Theta(\mathbf{x}-\mathbf{x}')n_\beta(\mathbf{x}')]c_\alpha(\mathbf{x})$. Here $\Theta(\mathbf{x})$ is the angle of the vector $\mathbf{x}$ in the plane [spatial (2D) vectors are indicated in boldface], the matrix $K=2\sigma^1$ [we denote the Pauli matrices $\sigma^\mu=(\sigma^z,\sigma^x,\sigma^y)$ for $\mu=0,1,2$], and $n_\alpha = c^\dagger_\alpha c_\alpha$. In terms of the ψ variables, standard techniques give the Euclidean Lagrange density for the system $\mathcal{L}=\mathcal{L}_\psi + \mathcal{L}_a$, with

$$\mathcal{L}_\psi=\overline{\psi}_\alpha(\partial_0-\mu-i\tilde{a}^\alpha_0)\psi_\alpha+\frac{1}{2m^*}|(\partial_j-i\tilde{a}^\alpha_j)\psi_\alpha|^2, \quad (3)$$

$$\mathcal{L}_a=\frac{i}{4\pi}K^{-1}_{\alpha\beta}\epsilon^{\mu\nu\lambda}a^\alpha_\mu\partial_\nu a^\beta_\lambda, \quad (4)$$

where Greek and Latin subscripts indicate three vector and two vectors, respectively $[\partial_\mu=(\partial_\tau,\boldsymbol{\nabla}),\ \partial_i=\boldsymbol{\nabla}]$, a^α_μ comprise a pair of Chern-Simons (CS) gauge fields, and μ is the chemical potential (usually taken positive). We use the notation that a gauge field with a tilde indicates the difference of CS and external gauge fields, e.g., $\tilde{a}^\alpha_\mu=a^\alpha_\mu-A^\alpha_\mu$, where A^α_μ is an external gauge field used both to include the magnetic field and for generating correlation functions by differentiation. At the mean-field level, $\langle\tilde{a}^\alpha_\mu\rangle=0$ at $\nu=1$—we consider fluctuations about this limit. In Eq. (4), we have dropped a Coulomb interaction term which will turn out to be irrelevant for the qualitative physics within the 111 state—it will be included when we return to the unpaired MCF liquid below. Instead, we assume for the moment that the interactions between MCF's (from both Coulomb and gauge sources) are such that they favor a pSC.[6,11] The BCS reduced Hamiltonian contains the additional term

$$H_\Delta=\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}[\Delta_{\boldsymbol{k}-\tilde{\boldsymbol{a}}^s}\psi^\dagger_{\boldsymbol{k}\uparrow}\psi^\dagger_{-\boldsymbol{k}\downarrow}+\text{H.c.}], \quad (5)$$

where $\psi_{\boldsymbol{k}\alpha}=\int d^2\boldsymbol{x}e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\psi_\alpha(\boldsymbol{x})$. In a pSC (more precisely, an $L^z=0$ triplet state) the pair field $\Delta_{\boldsymbol{k}}=e^{i\varphi}v(k^2)(k_x+ik_y)$, where φ is the phase of the pair wave function, and $v(k^2)$ is a smooth function of its argument. For simplicity, we will take $v(k^2)=v$ constant, adequate for *universal* properties. In the wavevector-dependent gap in Eq. (5), we have made the replacement $\boldsymbol{k}\to\boldsymbol{k}-\tilde{\boldsymbol{a}}^s$, with $a^{c/s}_\mu\equiv(a^\uparrow_\mu\pm a^\downarrow_\mu)/2$ (and similarly for $A_\mu,\tilde{a}_\mu$). Essentially, this follows from gauge invariance: the flux attachment introduces an unphysical gauge symmetry, but the Hamiltonian, being physical, must remain invariant under this. "Microscopically," this can be understood by deriving Eq. (5) via the usual mean-field decoupling of some density-density interaction. Because the density operator is automatically gauge invariant (i.e., the density of MCF's is equal to the electron density), one actually has a choice of whether to decouple by defining a gauge-invariant order parameter [including the gauge field as in Eq. (5)] or not (not including the gauge field). However, Elitzer's theorem tells us that there can be no local order parameter that spontaneously breaks the U(1) gauge symmetry. Hence the latter choice, without including the gauge field, will never lead to a consistent mean-field expectation value if gauge fluctuations are properly taken into account. Thus one is forced to include the gauge field in the pairing term. One can guess this intuitively by noting that the z's and w's in Eq. (2) are electron (not MCF) coordinates.

We then perform the key step of a combined particle-hole transformation and phase rotation

$$\Psi_\uparrow=e^{-i\varphi/2}\psi_\uparrow,\quad \Psi_\downarrow=e^{i\varphi/2}\psi^\dagger_\downarrow. \quad (6)$$

The phase rotation in Eq. (6) apparently "neutralizes" the Ψ fermions. Note, however, that the transformation becomes double valued in the presence of $\pm2\pi$ vortices in φ, which has important consequences to which we will return shortly. Equation (5) can then be reexpressed in Lagrangian form. Combining it with Eqs. (3)–(4) gives $\mathcal{L}=\mathcal{L}_\Psi+\mathcal{L}_a+\mathcal{L}_{\text{irr}}$, with

$$\mathcal{L}_\Psi=\overline{\Psi}[\partial_0-i\tilde{a}^s_0+iv\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}-i\tilde{\boldsymbol{a}}^s)-\mu\sigma^z]\Psi, \quad (7)$$

$$\mathcal{L}_{ac}=\frac{i}{4\pi}\epsilon^{\mu\nu\lambda}a^c_\mu\partial_\nu a^c_\lambda,\quad \mathcal{L}_{as}=-\frac{i}{4\pi}\epsilon^{\mu\nu\lambda}a^s_\mu\partial_\nu a^s_\lambda, \quad (8)$$

$$\mathcal{L}_{\text{irr}}=(\partial_\mu\varphi-2\tilde{a}^c_\mu)\mathcal{J}^\mu+\frac{1}{8m^*}|\boldsymbol{\nabla}\varphi-2\tilde{\boldsymbol{a}}^c|^2\overline{\Psi}\sigma^0\Psi - \frac{1}{2m^*}\overline{\Psi}\sigma^0(\boldsymbol{\nabla}-i\tilde{\boldsymbol{a}}^s)^2\Psi. \quad (9)$$

Here $\mathcal{L}_a=\mathcal{L}_{ac}+\mathcal{L}_{as}$ and $\mathcal{J}^0=\overline{\Psi}i\sigma^0\Psi/2$, $\boldsymbol{\mathcal{J}}=i[\overline{\Psi}(\boldsymbol{\nabla}-i\tilde{\boldsymbol{a}}^\sigma)\Psi-(\boldsymbol{\nabla}+i\tilde{\boldsymbol{a}}^\sigma)\overline{\Psi}\Psi]/2m^*$.

An effective field theory is obtained by "coarse graining"—i.e., integrating out gapped fermion modes at large $\mathbf{k}$. This reduces the ultraviolet momentum cutoff to Λ and also generates a "kinetic" term for the charge sector

$$\mathcal{L}_\varphi=\frac{n_{\text{SF}}}{2mv^2}(\partial_0\varphi-2\tilde{a}^c_0)^2+\frac{n_{\text{SF}}}{2m}(\boldsymbol{\nabla}\varphi-2\tilde{\boldsymbol{a}}^c)^2. \quad (10)$$

In the theory with the reduced cutoff, power-counting can be applied. The terms in $\mathcal{L}_{\text{irr}}$ [Eq. (9)] are irrelevant. They hence lead only to quantitative renormalizations of the effects described by $\mathcal{L}_\Psi+\mathcal{L}_a$, and will be suppressed hereafter.

Doing so, the Lagrangian becomes explicitly spin-charge separated. The charge sector is governed by $\mathcal{L}_c=\mathcal{L}_\varphi+\mathcal{L}_{ac}$, offering a physical interpretation as charge $2e$ composite bosons (Cooper pairs) at an effective filling factor $\nu_{\text{eff}}=1/4$. With the (4×4) conductivity tensor in the usual basis defined by $E^\alpha_i=\rho^{\alpha\beta}_{ij}J^\beta_j$, it is convenient to introduce charge and spin conductivities $\sigma^{c/s}_{ij}=2(\sigma^{\uparrow\uparrow}_{ij}\pm\sigma^{\uparrow\downarrow}_{ij})$. The (charge) Hall conductivity is quantized to $\sigma^c_{xy}=e^2/h=1/2\pi$ (in our units), as seen by choosing the gauge $\varphi=0$ and integrating out a^c_μ to obtain a CS term for A^c_μ. The spin sector Lagrangian is

$\mathcal{L}_s = \mathcal{L}_\Psi + \mathcal{L}_{as}$, describing massive Dirac fermions coupled to a spin CS gauge field. To analyze $\mathcal{L}_s$, we integrate out the Ψ fermions, which generates for $\mu > 0$ a CS term and nominally irrelevant Maxwell and higher-order in gradient corrections for $\tilde{a}^s_\mu$. Therefore the *effective* Lagrangian for the spin sector $\mathcal{L}_s \to \mathcal{L}_s^{\text{eff}}$ is

$$\mathcal{L}_s^{\text{eff}} = \frac{i}{4\pi}\epsilon^{\mu\nu\lambda}\tilde{a}^s_\mu \partial_\nu \tilde{a}^s_\lambda + \frac{1}{2\lambda}(\tilde{e}_j^2 - \tilde{b}^2) + O[\partial^3 \tilde{a}^2] + \mathcal{L}_{as}, \tag{11}$$

where $\lambda \sim (v^2/(\pi\mu) + \lambda_0^{-1})^{-1}$ is a nonuniversal "dielectric" constant, and $\tilde{e}_j = v^{-1}(\partial_j \tilde{a}^s_0 - \partial_0 \tilde{a}^s_j)$, $\tilde{b} = \epsilon_{ij}\partial_i \tilde{a}^s_j$. The λ_0 contribution to λ arises from integrating out the high-energy modes well above the gap, and is approximately v independent (though it does depend upon $\mathcal{L}_{\text{irr}}$). Significantly, the coefficient of $1/4\pi$ in the CS term above is a factor of two *larger* than what might naively (and incorrectly) be expected from the massive Dirac fermion in Eq. (7). This is a consequence of the fact that σ^s_{xy} is a nondissipative quantity, and generally dependent on not only the low-energy excitations of a system. Indeed, in a Kubo calculation, the proper result requires taking into account the decay of the superconducting gap $\Delta_{\mathbf{k}}$ at *large* momentum.[12] In any case, the $1/4\pi$ is a robust *topological* feature of the $k_x + ik_y$ state, and is independent of any approximations.

Note that upon combining all the terms in $\mathcal{L}_s^{\text{eff}}$, there is a cancellation of CS contributions for the fluctuating field a^s_μ (but not the external field A^s_μ). Now integrating out a^s_μ gives

$$\mathcal{L}_s^{\text{eff}} = \frac{n^s_{\text{SF}}}{2}[(A^s_j)^2 + v^{-2}(A^s_0)^2] + \frac{i\sigma^s_{xy}}{2}\epsilon^{\mu\nu\lambda}A^s_\mu \partial_\nu A^s_\lambda, \tag{12}$$

where $n^s_{\text{SF}} = \lambda v^2/4\pi^2$ is a spin superfluid density (stiffness), so that this state is a pseudospin superconductor—the QHFM. Interestingly, this state also exhibits a hidden spin Hall effect. The spin Hall conductance from Eq. (12) is $\sigma^s_{xy} \neq \hbar^2/h = \hbar/2\pi = 1/2\pi$, the lack of quantization of σ^s_{xy} being due to corrections from the nonuniversal $O[\partial^3\tilde{a}^2]$ terms in Eq. (11). The nonuniversality of σ^s_{xy} is perhaps natural since the U(1) symmetry generated by S^z is spontaneously broken.

Next consider the quasiparticle structure. For charge $2e$ bosons at $\nu_{\text{eff}} = 1/4$, the quasiparticle excitations, which correspond to the (smallest) 2π vortices in φ, carry the charge $\nu_{\text{eff}} \times 2e = e/2$, as can also be deduced directly from Eq. (10). Remarkably, owing to the implicit coupling in Eq. (6), this excitation also carries spin. In particular, the Ψ fermions experience a cut (π flux) upon encircling the vortex. Because the XY spin operator $S^+ \sim \Psi^\uparrow \Psi^\downarrow$ is bilinear in fermions, the charge $e/2$ quasiparticle is thus tied to a 2π spin-flux vortex (in S^+) (see also below). Moreover, because of the spin Hall conductivity, this flux induces a nonuniversal moment $\langle S^z \rangle = \pm \pi\sigma^s_{xy}$. We identify this excitation with the meron of Ref. 13 (the moment arises in that picture from pseudospin canting in the meron's core). Even-flux vortices in φ and S^+ leave Eq. (6) single-valued, and remain spin-charge separated. Due to the Higgs phenomena, the $\pm 4\pi$ vortices in φ are screened, cost finite energy, and therefore unbind at any finite temperature, giving activated contributions to the Hall effect. The merons, however, are tied to spin vortices which interact logarithmically and therefore exhibit a Kosterlitz-Thouless transition at finite temperature.

We now turn to the connection of the above formalism to the pseudospin magnetization approach of Refs. 13. To do so, we return to $\mathcal{L}_s = \mathcal{L}_d + \mathcal{L}_{as}$ [Eqs. (7),(8)]. Following the reasoning of Ref. 10, we argue that the CS gauge field "bosonizes" the Dirac fermions into relativistic charged bosons. As in the much more established (1+1)-dimensional bosonization mapping, the expressions for currents in terms of bosons are much simpler than those for the fermion fields, and we are presently unable to derive the latter. Instead, we will determine the form of the "bosonized" Lagrangian by requiring that it produce the same generating function for current-current correlators. Thus we seek an equivalent representation for the partition function

$$\mathcal{Z}_\sigma[A^s_\mu] = \int [d\bar{\Psi} d\Psi][da^s_\mu] e^{-\int d^3x_\mu \mathcal{L}_s} = e^{-\int d^3x_\mu \mathcal{L}_s^{\text{eff}}[A^s_\mu]}. \tag{13}$$

Referring back to Eq. (12), we recognize that the first term in $\mathcal{L}_s^{\text{eff}}$ is readily obtained from the usual "Higgs" mechanism if A^s_μ is minimally coupled to a U(1) boson which condenses. To reproduce the second (Chern-Simons) term in $\mathcal{L}_s^{\text{eff}}$ we introduce in addition a massive Dirac field which also carries the U(1) "charge" (actually spin). Thus

$$\mathcal{Z}_\sigma[A^s_\mu] = \int [d\bar{\eta} d\eta][d\theta] e^{-\int d^3x_\mu \mathcal{L}_s^{\text{dual}}}, \tag{14}$$

where

$$\mathcal{L}_s^{\text{dual}} = \frac{n^s_{\text{SF}}}{2mv^2}(\partial_0\theta - 2A^s_0)^2 + \frac{n^s_{\text{SF}}}{2m}|\nabla\theta - 2\mathbf{A}^s|^2 + \bar{\eta}[\partial_0 - iA^s_0 + iv\boldsymbol{\sigma}\cdot(\nabla - i\boldsymbol{A}^s) - M\sigma^z]\eta + \gamma[e^{i\theta}\bar{\eta}\sigma^y\bar{\eta} + \text{H.c.}]. \tag{15}$$

From Eq. (15), we identify $S^+ \sim e^{i\theta}$. In Eq. (15), in addition to a Dirac Lagrangian of the usual form, we have also included an "anomalous" coupling which exchanges the spin between the θ boson and η fermions. Given only the single physical U(1) spin-rotation symmetry, such a coupling is allowed and indeed is required to reproduce the *unquantized* spin Hall conductivity in Eq. (12).

Having established and explored the equivalence of the paired MCF state and the QHFM, we now turn to a discussion of possible "quantum disordered" ground states suggested by this work. Specifically, we consider cases in which the (charge) Hall resistance is unquantized, motivated by the experimental observation of poorly developed Hall plateaus. These phases can be described loosely by the proliferation in the ground state of charge vortices, i.e., point defects around which $\oint \vec{\nabla}\varphi \cdot d\vec{r} = 2\pi N$, with integer N. The two possible phases of interest correspond to the cases in which (i) only vortices with even N proliferate, leaving θ single valued and (ii) all vortices are unbound.

The ''doubly quantized'' vortices in case (i) are conventional, insofar as they leave the singular gauge transformation in Eq. (6) single valued. Hence these vortices interact only weakly with the other excitations of the system. Their proliferation can thus be analyzed using conventional methods. In particular, by performing a (2+1)-dimensional duality transformation on the XY model in Eq. (10), the proliferated state can be described as a condensate of pairs (due to the even N condition) of vortices (merons). (Dual) phase coherence of the vortex-pair wave function implies the quantization of charge in units of half the composite boson charge or e. Thus the bilayer charge density of e per area l^2 is distributed into a Wigner crystal of charge e per unit cell. This charge e Wigner crystal is of course an electrical insulator (with $\sigma^c_{xx},\sigma^c_{xy}\to 0$ as $T\to 0$ provided the sliding mode is even infinitesimally pinned). Because the paired vortex condensate respects spin-charge separation, however, interlayer phase coherence (spin superfluidity) is maintained. An alternative picture for this phase is as a staggered bilayer crystal in which vacancy-interstitial pairs made from opposite layers have Bose condensed. Thus the (pseudo)spin conductances are very different: $\sigma^s_{xx}(\omega)=n^s_{\mathrm{SF}}/i\omega$, so that the zero-bias spin conductivity is infinite, while σ^s_{xy} is a nonuniversal constant. Note that this implies $\rho^s\to 0$ for $T\to 0$, so the single-layer resistivity $\rho^{\uparrow\uparrow}\approx\rho^c$ does not manifest superfluidity. At strictly zero temperature, $\rho^c_{xx}=\infty$, so that the drag resistance would diverge. For $T>0$, it is natural to expect a peak in $\rho^{\uparrow\uparrow}_{xx}$ as a function of d/l as a precursor effect. Because of interlayer phase coherence, however, the charge e Wigner crystal should exhibit a zero-bias tunneling conductance peak as in the QHFM.

In case (ii), by contrast, the strong statistical interaction of individual merons amongst themselves and with other excitations renders their proliferation a strong-coupling problem. On physical grounds, however, we speculate that their presence in the ground state simply destroys all effects of MCF pairing on long length and time scales. We are therefore led to consider a simple model of MCF's without pairing, analogous (but distinct from) the composite Fermi liquid. This MCF Liquid is described simply by the Lagrangian $L=\int d^2\vec{x}[\mathcal{L}_\psi+\mathcal{L}_a]+L_C$, where $\mathcal{L}_\psi$ and $\mathcal{L}_a$ are given in Eqs. (3),(4), and due to the nonvanishing MCF compressibility, it is necessary a priori to include an additional long-range Coulomb interaction term L_C (see Refs. 14–16).

Drag between *weakly coupled* layers in nearly independent $\nu=1/2$ composite Fermi liquid states has been considered previously by many authors.[14,16] In that case, the drag *resistivity* $\rho^{\uparrow\downarrow}$ is truly perturbative in the interlayer interaction., and the formalism due originally to Zheng and MacDonald[17] can be applied to yield a small longitudinal drag resistivity $\rho^{\uparrow\downarrow}_{xx}\sim T^{4/3}$ at low temperature.[16] The inherently strong interlayer interactions in the MCF liquid unfortunately preclude this approach, and one is reduced to a diagrammatic treatment as in Refs. 14,15. This diagrammatic treatment is much less satisfactory, but reasoning along the lines of Refs. 14,15 suggests, and we therefore propose, that $\rho^{\uparrow\downarrow}_{xx}\sim T^{4/3}$ also obtains for the MCF liquid. Unlike, the ICFL, however, the random phase approximation[5] already gives a nonzero ''Hall drag'' $\rho^{\uparrow\downarrow}_{xy}=4\pi$, and we expect this is robust. Thus the longitudinal drag resistivity is small also in this case, and only the Hall drag is expected to deviate substantially from the ICFL limit. Furthermore, the interlayer tunneling conductance in the MCF liquid, similar to that of the ICFL state, is expected to exhibit a pseudogap due to orthogonality catastrophe and poorly screened Coulombic effects.

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