Fermionic Paired Superfluids in Rotating Trap

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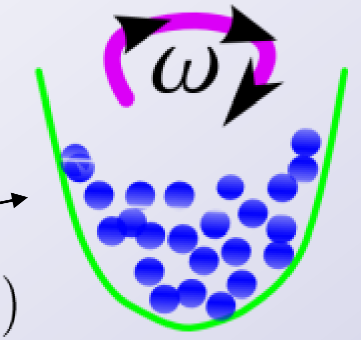
Outline

- Motivation:
  - Effect of rotation on degenerate Fermi gas across Feshbach resonance
    (Experiments with ^{6}Li [Zweirlen et al. Nature, 2005])
  - Fermions vs. Bosons:
    Additional mechanisms supress Tc for fermions (Gorkov).
- Two-Channel Model In a Rotating Trap
  - Model
  - Perturbative Expansion
- Results
  - Non-Rotating Trap
  - Rotating Trap + Scaling
- Future Work

Two Channel Model:
Fermion +Boson in Rotating Trap



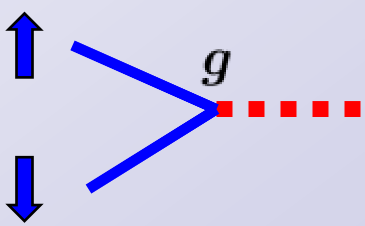
$$V(\mathbf{r}) = \frac{m}{2}\left(\Omega_\perp^2(x^2+y^2) + \Omega_z^2 z^2\right)$$

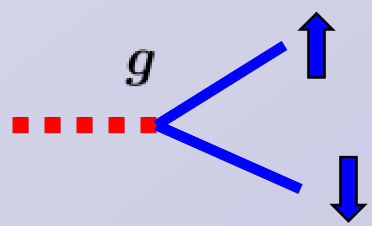
In Rotating Frame: $\mathcal{H} = \mathcal{H}^{(F)} + \mathcal{H}^{(B)} + \mathcal{H}^{(I)}$

$$\mathcal{H}^{(F)} = \sum_\sigma \int d^3\mathbf{r}\, \psi_\sigma^\dagger(\mathbf{r}) \left[-\frac{\hbar^2\nabla_\mathbf{r}^2}{2m} + V(\mathbf{r}) - \mu - \omega L^z\right] \psi_\sigma(\mathbf{r})$$

$$\mathcal{H}^{(B)} = \int d^3\mathbf{r}\, \phi^\dagger(\mathbf{r}) \left[-\frac{\hbar^2\nabla_\mathbf{r}^2}{2(2m)} + 2V(\mathbf{r}) - 2\mu + \delta - \omega L^z\right] \phi(\mathbf{r})$$

$$\mathcal{H}^{(I)} = g \int d^3\mathbf{r} \left(\phi^\dagger(\mathbf{r})\psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r}) + \psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r})\phi(\mathbf{r})\right)$$





Procedure for calculation of T_c (Perturbative expansion in g^2)

- Imaginary time action
- Integrate out Fermions
- Expand out Z to second order in Bosonic fields
- T_c given by Thouless criterion subject to constraint by particle number

$$Z = \int \mathcal{D}\phi \mathcal{D}\psi e^{-S}$$

$$S = \int d\tau d^3\mathbf{r} \left[\bar{\psi}_\sigma \left(-\left(\mathcal{G}_0^F\right)^{-1} \right) \psi_\sigma + \bar{\phi} \left(-\mathcal{G}_0^B \right)^{-1} \phi + g \left(\bar{\phi} \psi_\uparrow \psi_\downarrow + \phi \bar{\psi}_\downarrow \bar{\psi}_\uparrow \right) \right]$$

$$S = -Tr \log \begin{pmatrix} -\left(\mathcal{G}_0^F\right)^{-1} & g\phi \\ g\bar{\phi} & -\left(\mathcal{G}_0^F\right)^{-1} \end{pmatrix} + \int d\tau d^3\mathbf{r} \bar{\phi} \left(-\mathcal{G}_0^B \right)^{-1} \phi$$

$$S \approx -\log Z_0^F + \int d\tau d^3\mathbf{r} \bar{\phi}(\mathbf{r}) \left(-\mathcal{G}_0^B \right)^{-1} \phi(\mathbf{r}) - g^2 \int d\tau d^3\mathbf{r} d^3\mathbf{r}' \bar{\phi}(\mathbf{r}) \Pi(\mathbf{r},\mathbf{r}') \phi(\mathbf{r}')$$

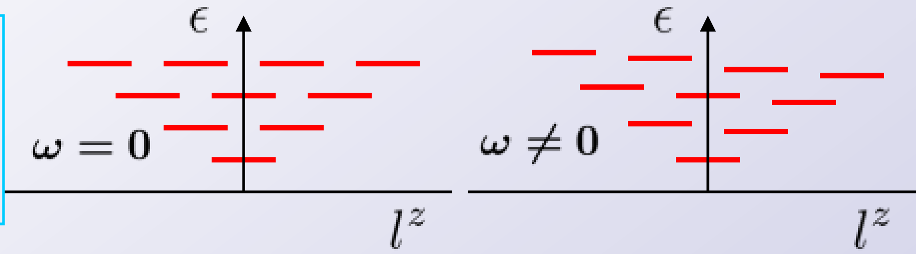
$$\Pi(\mathbf{r},\mathbf{r}') = \mathcal{G}_0^F(\mathbf{r},\mathbf{r}') \mathcal{G}_0^F(\mathbf{r},\mathbf{r}')$$

$$Min_\lambda \left[(\mathcal{G}_0^B)^{-1}(0) + g^2 \Pi(0) \right] = 0$$

$$N = N_0^F + 2N^B \text{ where } N^B = \frac{1}{\beta} \sum_n \frac{e^{i\Omega_n 0^+}}{(\mathcal{G}_0^B)^{-1}(i\Omega_n) + g^2 \Pi(i\Omega_n)}$$

Using Harmonic Oscillator Basis

$$\epsilon^F_{l=\{L,l^z,l_0\}} = \hbar\Omega_\perp(2L+1) + 2\hbar\omega l^z + \hbar\Omega_z l_0 - \mu$$
$$\epsilon^B_{l=\{L,l^z,l_0\}} = \hbar\Omega_\perp(2L+1) + 2\hbar\omega l^z + \hbar\Omega_z l_0 - 2\mu + \delta$$



Thouless Criterion+ Particle Number

①

$$\delta - 2\mu = \frac{g^2}{4\pi^{5/2}\bar{a}^3_{HO}} \sum_{l=\{L,l^z,l^0\}} \frac{e^{-\frac{2(l^z)^2}{L}}}{\sqrt{Ll^0}} \left[\frac{\tanh(\beta\epsilon^F_l/2) + \tanh(\beta\epsilon^F_{\bar{l}}/2)}{\epsilon^F_l + \epsilon^F_{\bar{l}}} - \frac{2}{\epsilon^F_l + \epsilon^F_{\bar{l}} + 2\mu} \right]$$

ω dependence

②

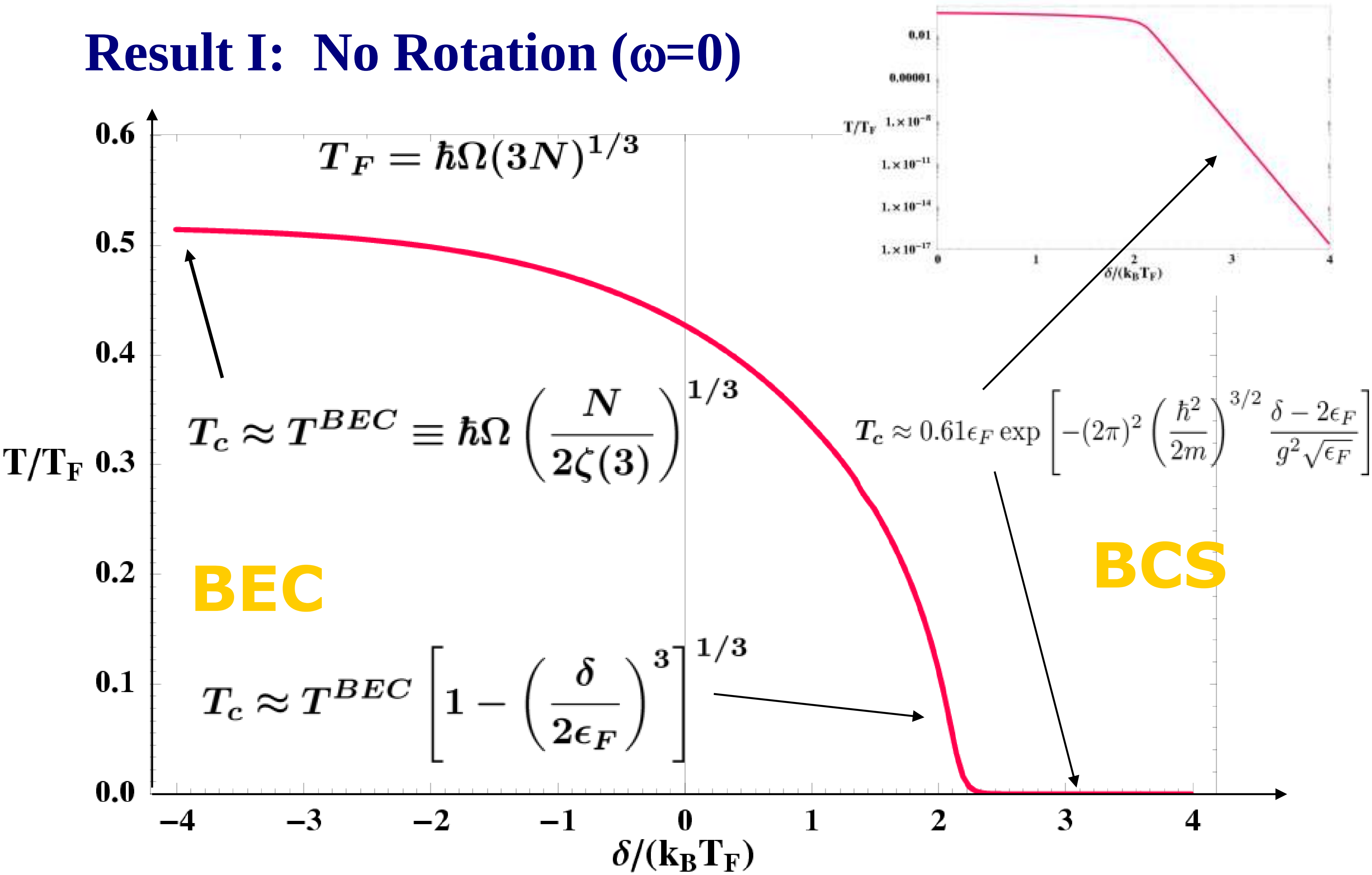
$$N = \frac{2}{(\beta\hbar\tilde{\Omega})^3}\left(Li_3\left[e^{\beta(2\mu-\delta)}\right] - Li_3\left[-e^{\beta\mu}\right]\right) + \mathcal{O}(g^2) \text{ where } \tilde{\Omega} = \Omega\left[1 - \left(\frac{\omega}{\Omega_\perp}\right)^2\right]^{1/3}$$

Solve for (μ, β)

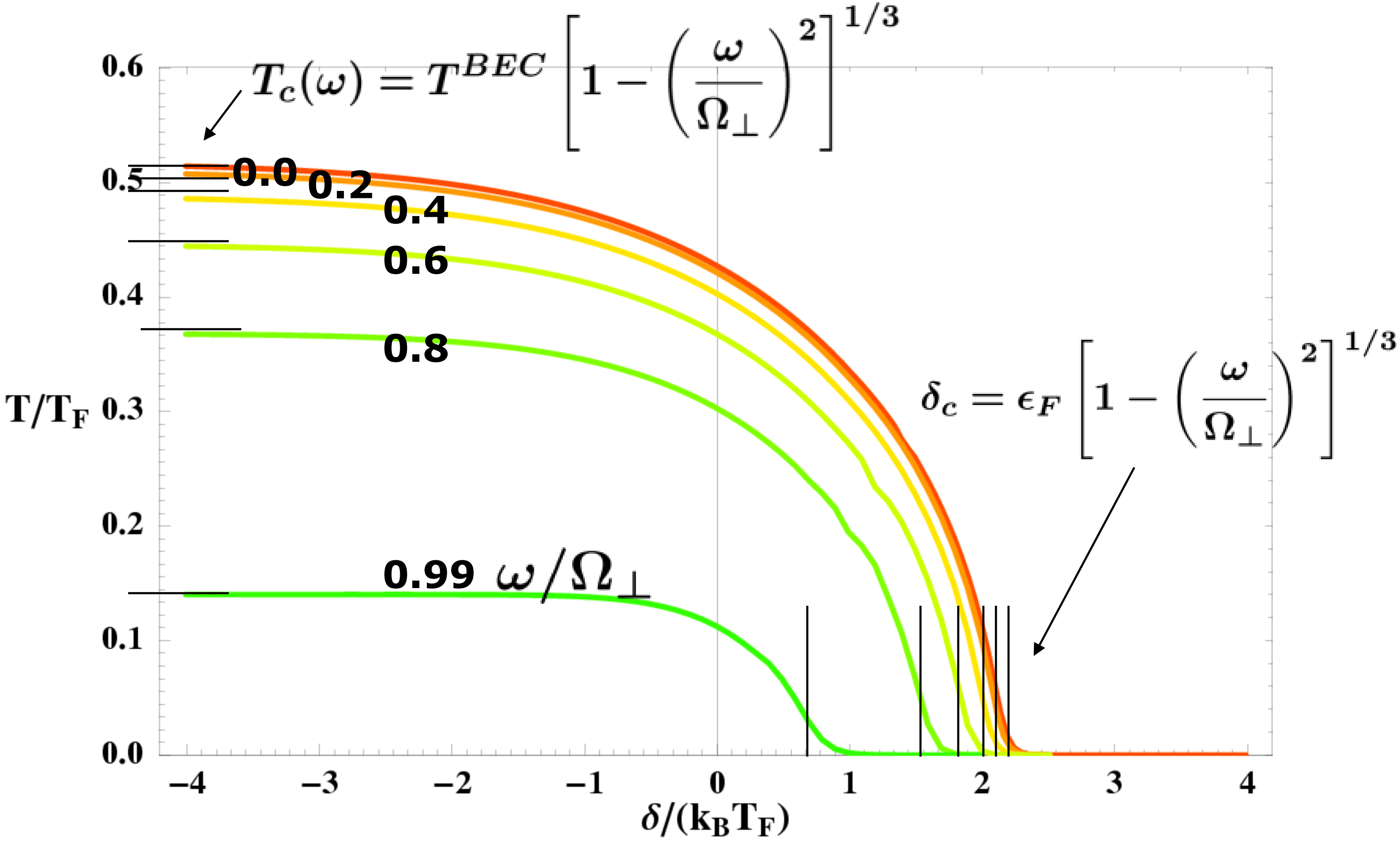
① **Depairing effect: (Gorkov)**
Rotation ⟺ Magnetic Fields in SC

② **Weakening of the trap:**
lowering of the local density

Result I: No Rotation (ω=0)



Result II: Rotating Trap



Result III

- Scaling under rotation

  in Zeroth order in g

$$\delta \to \delta \left[1 - \left(\frac{\omega}{\Omega_\perp}\right)^2\right]^{1/3}$$

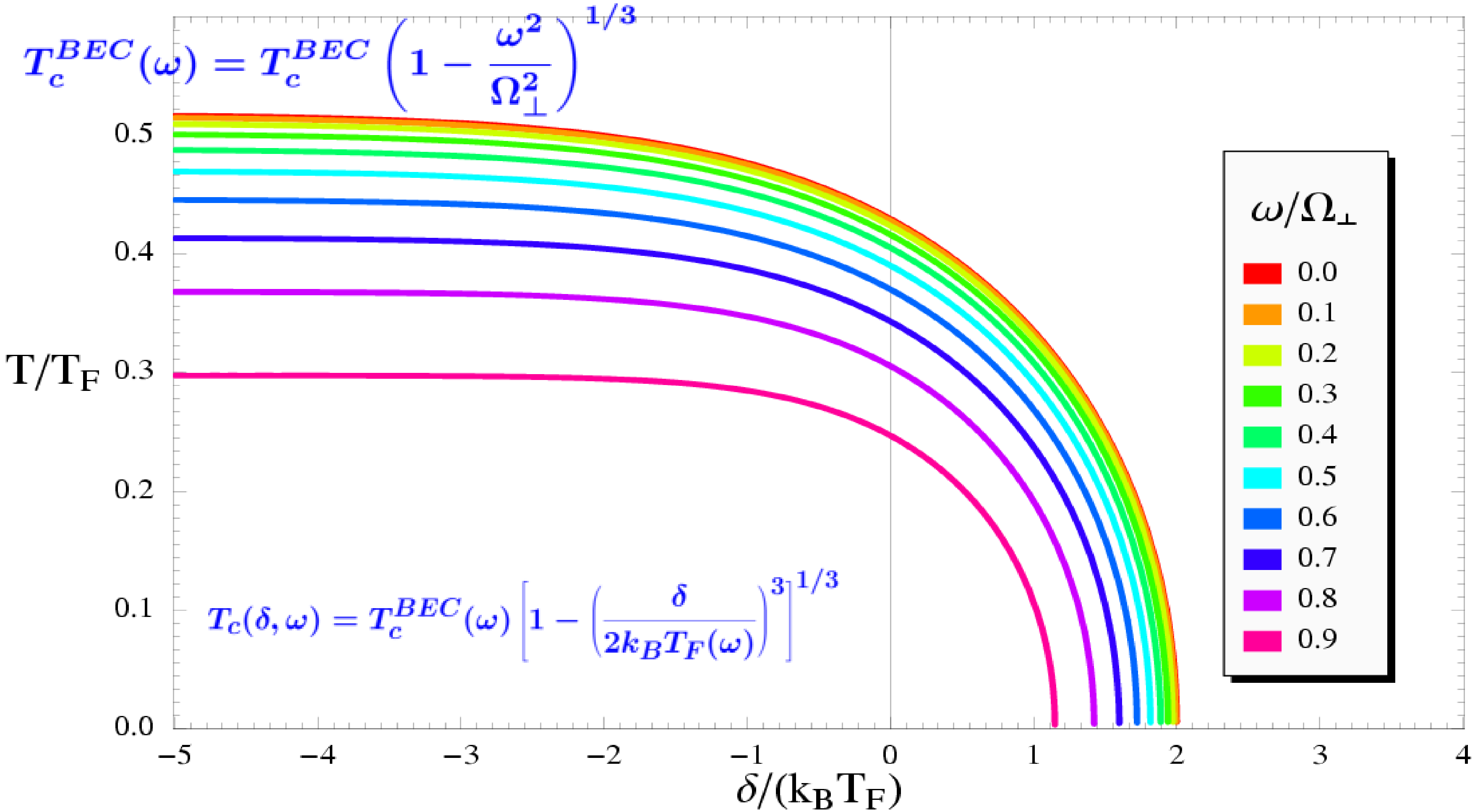
$$T \to T \left[1 - \left(\frac{\omega}{\Omega_\perp}\right)^2\right]^{1/3}$$

- Further corrections cannot be obtained by rescaling interactions

Eg: BCS regime

$$T_c = T_c^0 \left[1 - \left(\frac{\omega}{\Omega_\perp}\right)^2 \left\{\frac{1}{3} + \frac{2\pi^2(\delta + 2\epsilon_F)}{3g^2\sqrt{\epsilon_F}} \left(\frac{\hbar^2}{2m}\right)^{3/2} + \frac{21}{2^{3/2}\pi^{7/2}} \frac{(\beta_c \epsilon_F)^2}{\log[\beta_c \epsilon_F]} \frac{\hbar \Omega_\perp}{\epsilon_F}\right\}\right]$$

Non-interacting Phase Diagram $T_F(\omega) = T_F^0\left(1 - \frac{\omega^2}{\Omega_\perp^2}\right)^{1/3}$



Symmetries of Hamiltonian

- Conservation of Particle Number

$$N = \int d^3\mathbf{r} \left(2\langle \phi^\dagger(\mathbf{r})\phi(\mathbf{r})\rangle + \sum_\sigma \langle \psi^\dagger{}_\sigma(\mathbf{r})\psi_\sigma(\mathbf{r})\rangle \right)$$

- Angular Momentum $[\mathcal{H}, L^z] = 0$
- Translation Symmetry in the x-y plane in the limit $\omega = \Omega_\perp$

Summary

- Investigate Critical Temperature of Fermion-Boson Model in a rotating trap
- Main effect of rotation
  - Lowering of T_c through a decrease of local density of atoms
- Future Work:
  - Derivation of G-L equation in rotating trap
  - Study $\omega = \Omega_{\perp}$ regime