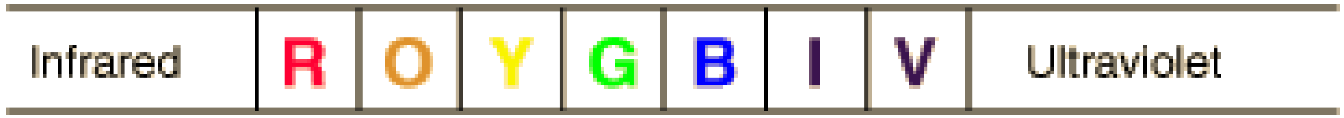
PHY 203: Light and Color



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Text: Light Science - Physics and Visual Arts

Grading:

HW	15%
Lab	15%
2 exams	40% (20% each)
Project/paper	15%
Final	15%

Attendance Policy:

Attendance is required to all lectures, exams, and lab exercises. No make-ups for anything, except in extreme cases of illnesses or approved college activities.

The material I am hoping to cover can be subdivided into four primary sections:

a. Nature of Light and Color
b. Geometrical Optics
c. Polarization
d. Fourier Optics

The above four areas are parts (1) and (2) of the study of Optics in general which can be subdivided into three areas:

1. Geometrical Optics
2. Physical Optics
3. Quantum Optics

a. Nature of Light and Color

We need to start somewhere………..
Let us start with the question "What is light?"

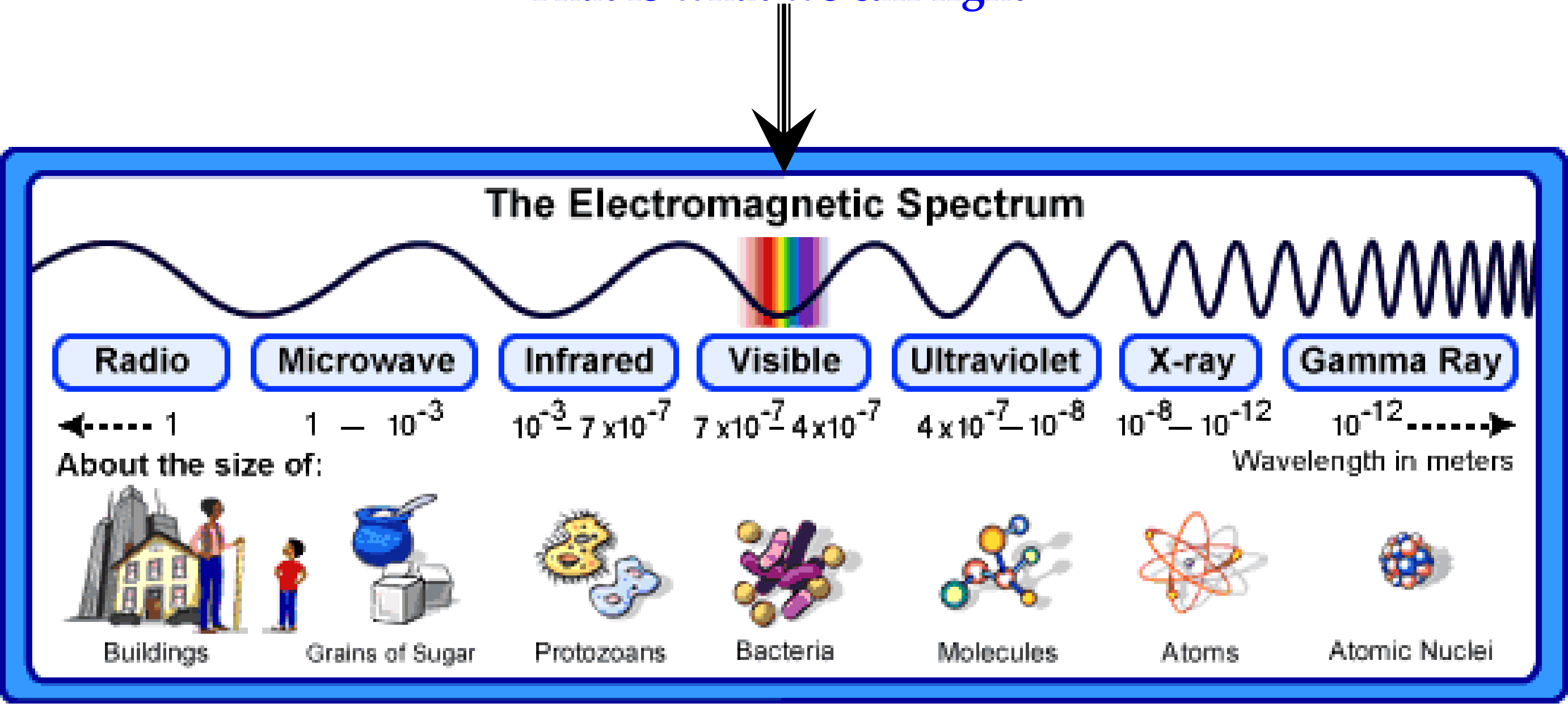
Light is a name for a range of electromagnetic radiation that can be detected by the human eye.

Okay then, What is electromagnetic radiation?

Electromagnetic radiation is radiation, you know …
That thing that can behave like particles and like waves.

The point here is that we describe things by their properties!
Just like the question have you seen an electron lately?

That is what we call light



$$Velocity\ .of\ .light\ .in.vacuum = wavelength\ .\ frequency$$

$$C = \lambda . f$$

Wave view of light:

One way to look at electromagnetic radiation (light) is as changing electric and magnetic fields which propagate through space, forming an **electromagnetic** **wave**. This wave has **amplitude**, which is the **brightness** of the light, ***wavelength***, which is the ***color*** of the light, and an angle at which it is vibrating, called **polarization**. This was the classical interpretation, crystallized in Maxwell's Equations.

Modern/particle view of light:

In terms of the modern quantum theory, electromagnetic radiation consists of particles called **photons**, which are packets ("quanta") of **energy** which move at the **speed of light**. In this particle view of light, the **brightness** of the light is **the number of photons**, the *color* of the light is the *energy* contained in each photon, and four Jones numbers (say X, Y, Z and T) are for **polarization**.

We still can divide the study of light into another subdivisions:

A) **Ray Optics (Geometrical Optics)**

B) **Wave Optics (Physical Optics)**

....

C) **Light as a Relativistic Object**

D) **Light as a Quantum Object**

A) Ray Optics (Geometrical Optics)

1) Rays satisfy the laws of reflection and refraction.

2) Rays are normal to wavefronts.

3) Two wavefronts are separated by the same optical path length when measured along any rays separating them.

4) The rays follow the path of least time (the optical path is stationary with respect to variables that define it).

   a) Light travels along straight lines in a homogeneous medium.

   b) Light travels the same path going either direction from A to B or B to A.

5) The ray density is proportional to the irradiance.

B) Wave Optics (Physical Optics)

1) Objects diffract light.

2) Light evidences interference.

3) Light carries energy and momentum.

Wave optics reduces to geometrical optics in the short wavelength limit. Practically this means geometrical optics works when critical dimensions are more than about 100 wavelengths.

C) Light as a Relativistic Object

1) All observers measure the speed of light to be the same regardless of their relative motion.

2) Light has zero rest mass.

$$E^2 = p^2c^2 + m^2c^4$$

$$E = pc \quad m = 0$$

3) In inertial frames light travels in a straight line but may curve in non-inertial frames..

4) Maxwell's equations, unlike the equations of classical mechanics, are Lorentz invariant.

D) Light as a Quantum Object

1) Light propagates like a wave but exchanges energy and momentum like a particle.

2) The De Broglie relations connect wave and particle properties.

$$E = \eta\omega = h\nu$$

$$p = \eta k = h/\lambda$$

Quantum electrodynamics is the most accurate theory known today. QED reduces to wave optics in the limit that h tends to zero. In this limit quanta cease to exist and energy and momentum become continuous rather than discrete.

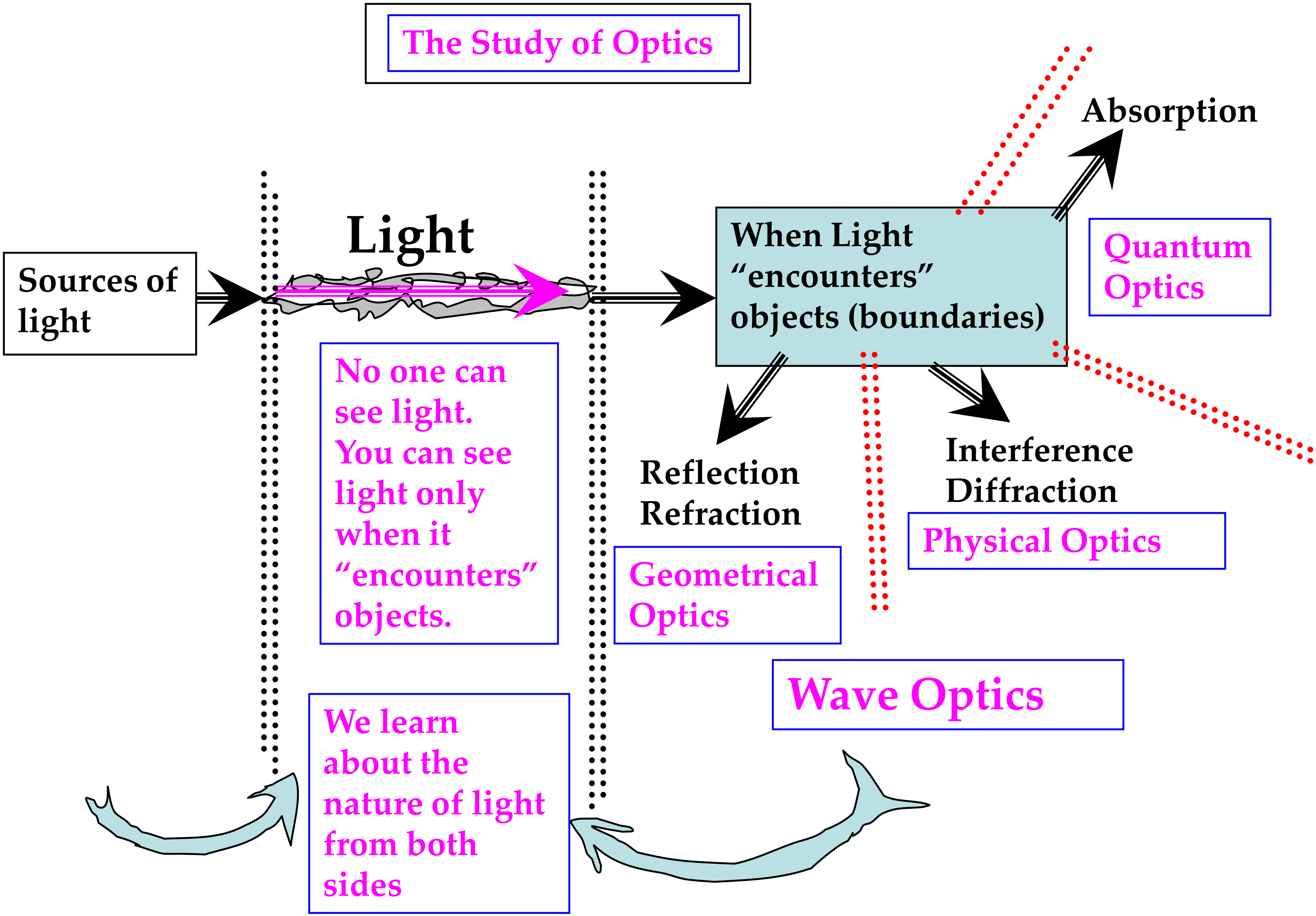
As you can see that there are many definitions and terms. These are all attempts to describe something that is hard to describe – Light.

But we can combine as many of these terms together as we can to answer as many questions as possible.

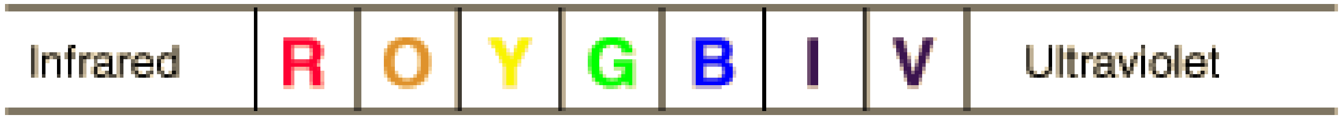
> A beam of sunlight (intense) that strikes a lens or prism or a set of slits, <u>wave theory</u> is adequate to calculate what fraction of light will go in any given direction.
>
> But if the beam is feeble and short in duration, the wave theory predict smooth distribution just like when the beam is intense, yet the distribution is granular (discrete). The <u>particle theory</u> is the one that explains this phenomenon accurately.

So we will need both approaches if we really want to understand the mystery of light. We will be concerned mostly with wave theory only.

Let us look at more definitions and terms …



The Study of Optics
Sources of light
Light
No one can see light. You can see light only when it "encounters" objects.
We learn about the nature of light from both sides
When Light "encounters" objects (boundaries)
Absorption
Quantum Optics
Reflection
Refraction
Geometrical Optics
Interference
Diffraction
Physical Optics
Wave Optics



400 nm .... 700 nm

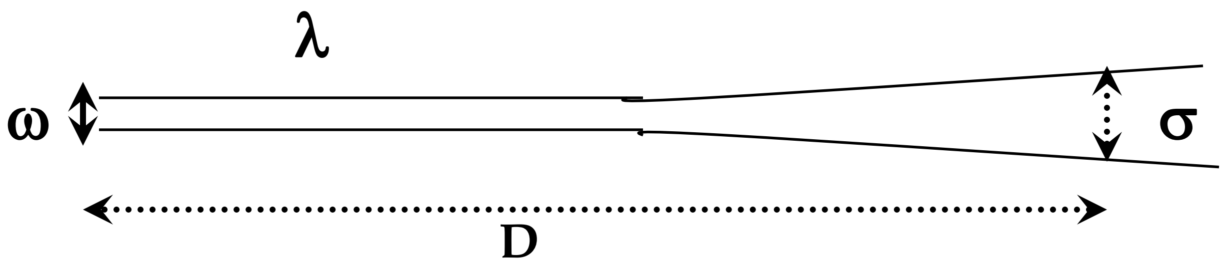
We want to work with the visible range i.e. wavelengths between 400 nm and 700 nm.

When Geometrical Optics is used?

Geometrical Optics is a good approximation when the light wavelength is much smaller than any dimension of the optical apparatus (usually centimeter or millimeter in size) used.

Here is a simple argument of calculating when Geometrical Optics is used

A light beam of wavelength λ and initial width ω, will, after a distance D, have spread σ such that they are related by: $\lambda . D = \omega . \sigma$



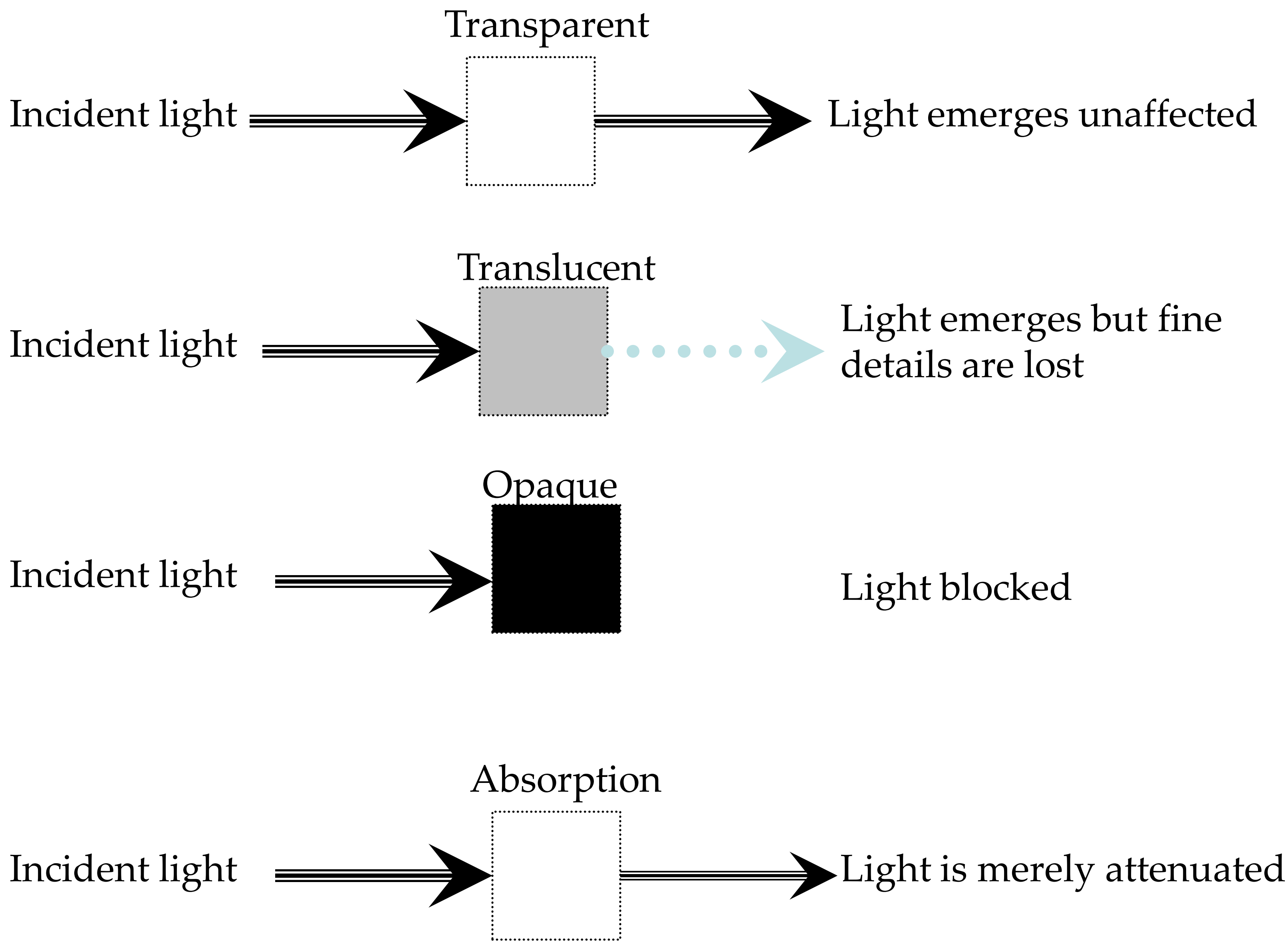
Actually you can guess the above relation just from size consideration (very large (D) x (very small (λ) = (average (ω) x average (σ). So the beam starts to spread is when σ ≥ ω or

$$D \geq \frac{\omega^2}{\lambda}$$

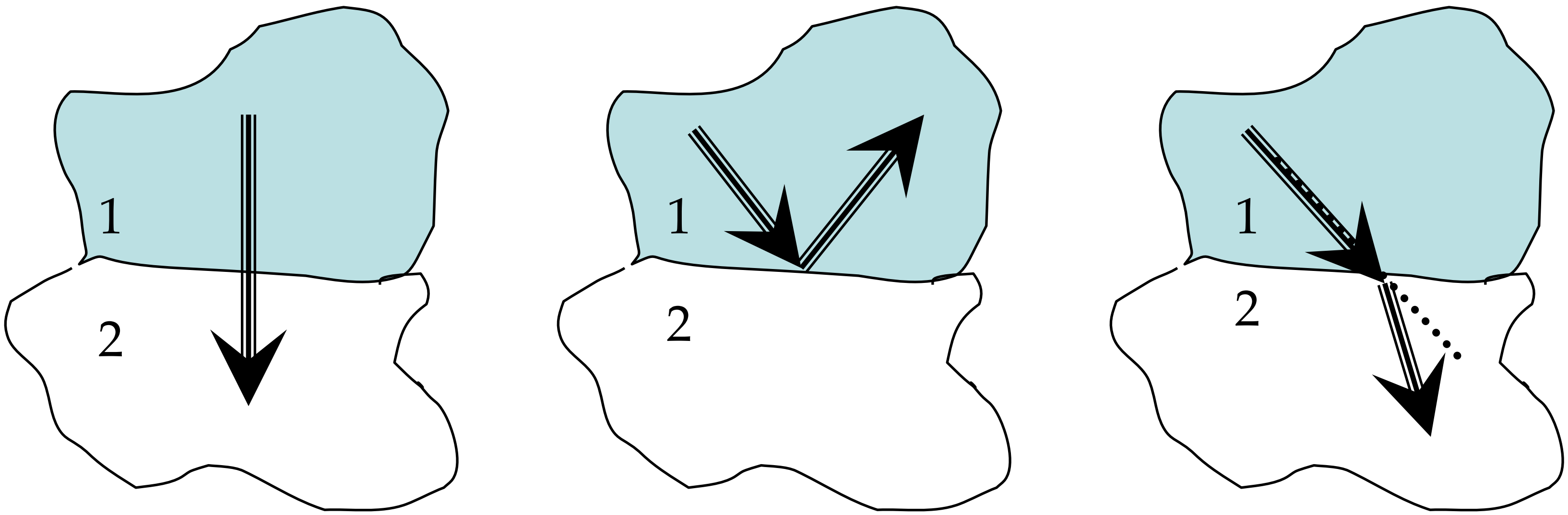
So even for beams of ω = 5 mm, and about 500 nm (visible light), the distance before the light starts to spread (i.e. the distance that it travels in **a straight line**), D is 50 m. This is a good distance to justify our approximation! So in a **homogenous medium** light travels in a straight line.

After this distance then we enter the Fraunhofer approximation where we need wave optics.

Four types of media



At a Boundary one or more of three things can happen ..



Light transmitted

Light reflected

Light refracted

Laws of Geometrical Optics

1. Rectilinear Propagation

2. Reflection

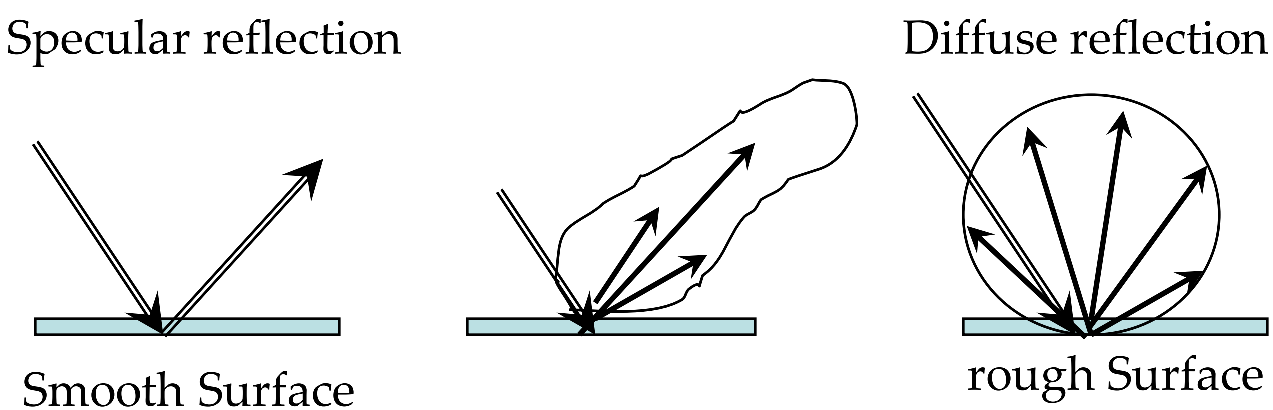
3. Refraction

All are derived from Fermat's principle

Fermat's principle: Light travels such that the time of travel is minimal – hence the "**principle of least time**"

Let us look at how reflection and refraction can be derived from Fermat's principle.

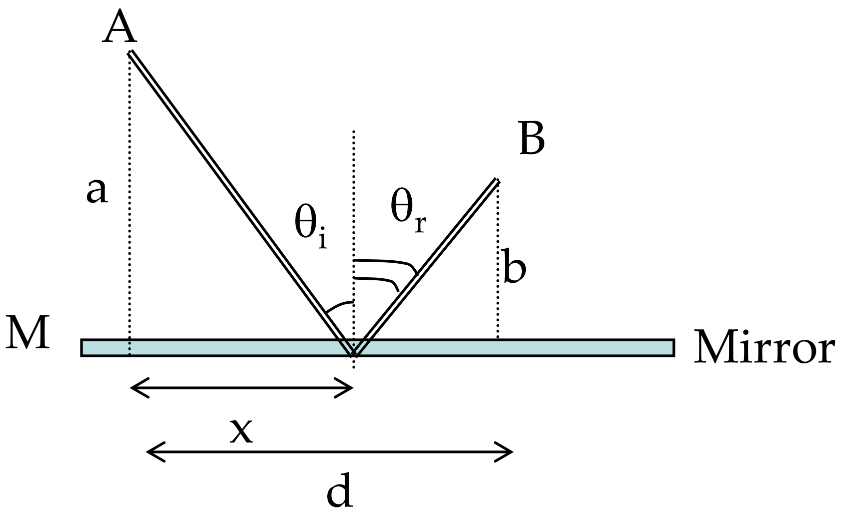
Reflection



Reflection

Smooth Surface

Specular reflection



Light coming from A and reflected from a mirror M toward B. The geometric length, s, is given by: $s = \sqrt{a^2 + x^2} + \sqrt{(d-x)^2 + b^2}$ and t = s/v

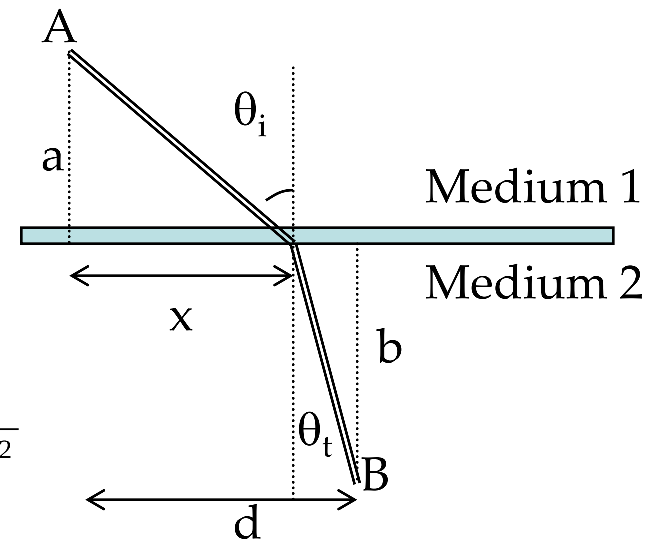
And if Fermat's principle hold true then the derivative of t with respect to x must be zero, dt/dx =(1/v) ds/dx = 0 or ds/dx = 0.

$$\frac{ds}{dx} = \frac{1}{2}\frac{1}{\sqrt{a^2+x^2}}2x + \frac{1}{2}\frac{1}{\sqrt{(d-x)^2+b^2}}2(d-x)(-1) = 0$$

Which reduces to $\frac{x}{\sqrt{a^2+x^2}} = \frac{d-x}{\sqrt{(d-x)^2+b^2}}$ or $\sin(\theta_i) = \sin(\theta_r)$ or $\theta_i = \theta_r$

The Law of Reflection: The angle of incidence equals the angle of reflection

Refraction



Light coming from A, encounters a boundary and refracted toward B. The geometric length is $s = s_1 + s_2$, and the total time of travel is:

$$t = \frac{s_1}{v_1} + \frac{s_2}{v_2} \qquad s_1 = \sqrt{a^2 + x^2} \qquad s_2 = \sqrt{(d-x)^2 + b^2}$$

And if Fermat's principle hold true then the derivative of total time of travel with respect to x must be zero, dt/dx = 0.

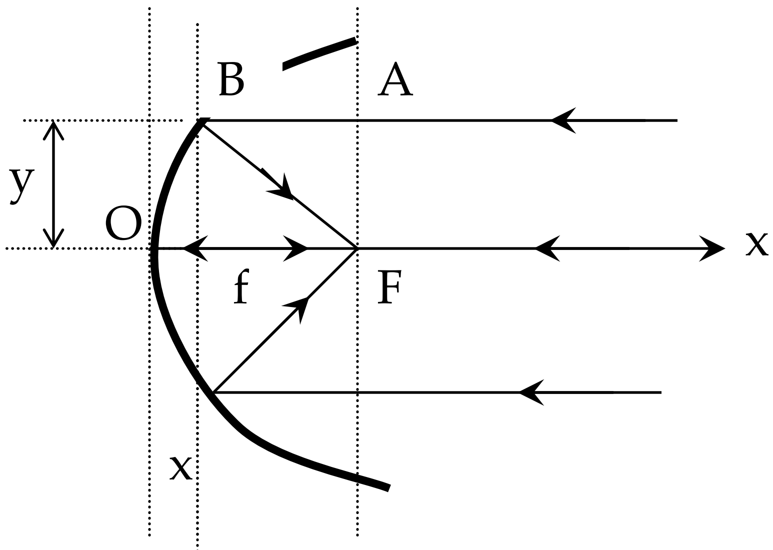
$$\frac{dt}{dx} = \frac{1}{v_1}\frac{1}{2}\frac{1}{\sqrt{a^2+x^2}}2x + \frac{1}{v_2}\frac{1}{2}\frac{1}{\sqrt{(d-x)^2+b^2}}2(d-x)(-1) = 0$$

Which reduces to $\frac{1}{v_1}\frac{x}{\sqrt{a^2+x^2}} = \frac{1}{v_2}\frac{d-x}{\sqrt{(d-x)^2+b^2}}$ or $\frac{1}{v_1}\sin(\theta_i) = \frac{1}{v_2}\sin(\theta_t)$

Using $v = \frac{c}{n}$ we will get $n_1 \sin(\theta_i) = n_2 \sin(\theta_t)$ n is called index of refraction.

The Law of Refraction: $n_1 \sin(\theta_i) = n_2 \sin(\theta_t)$ which it is called Snell's law.

Ideal Optical instruments is such that all the rays between the object and the image have equal optical paths. This implies Fermat's least time principle. So the method of calculating the shape of the ideal instrument (whatever the purpose may be) is the same. First choose two independent paths and calculate their respective times. Then require these time are the same. This will give us the desired shape of the instrument. Below we consider few cases, we start with the case of ideal mirror.



$$P_{FOF} = 2f$$

$$P_{ABF} = (f - x) + \sqrt{(f - x)^2 + y^2}$$

Equating the above two expressions

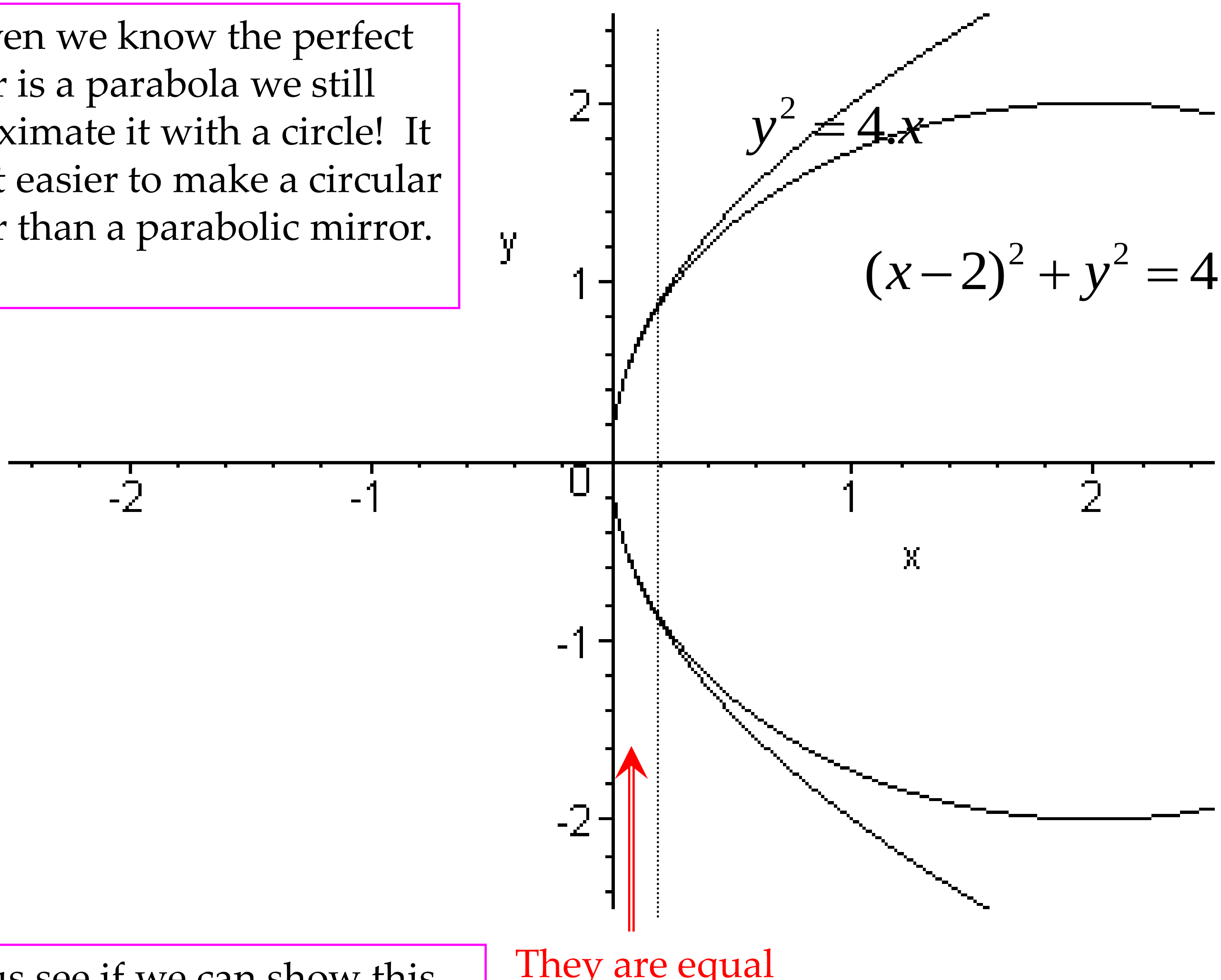
$$2f = \sqrt{(f - x)^2 + y^2} + (f - x)$$

$$(f + x)^2 = (f - x)^2 + y^2$$

$$y^2 = 4f.x$$ An equation of a parabola

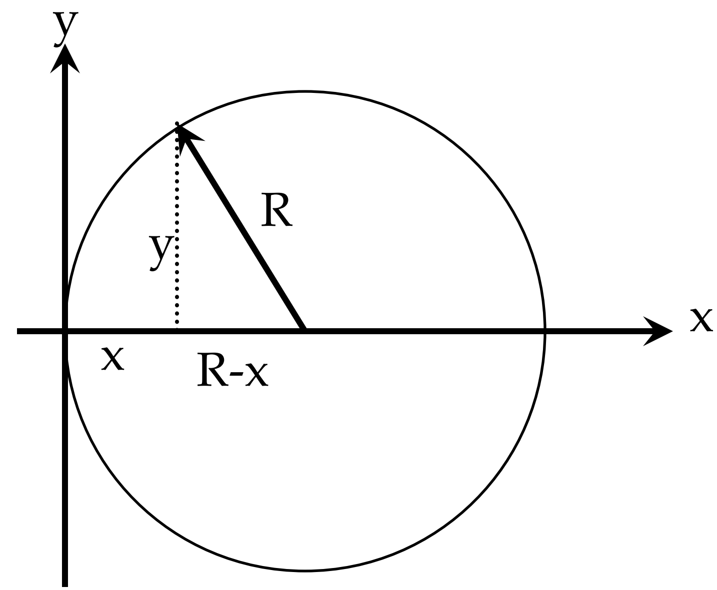
So the perfect mirror is a parabola, not a circle!

But even we know the perfect mirror is a parabola we still approximate it with a circle! It is a lot easier to make a circular mirror than a parabolic mirror.



Let us see if we can show this approximation in a more convincing way.

Small angle approximation here is equivalent to that we do not deviate from the optical axis by more than 15.



This approximation can be shown as:

$$y^2 + (R - x)^2 = R^2$$

$$y^2 + R^2 - 2Rx + x^2 = R^2$$

$$y^2 - 2Rx \cong 0$$

$$x = \frac{y^2}{2R}$$

$$y^2 = 4f.x$$

Which is the previous equation of a parabola!

We will use $x = \frac{y^2}{2R}$ again

late when we look at lenses.

Or using the binomial expansion

$$y^2 + (R - x)^2 = R^2$$

$$(R - x)^2 = R^2 - y^2$$

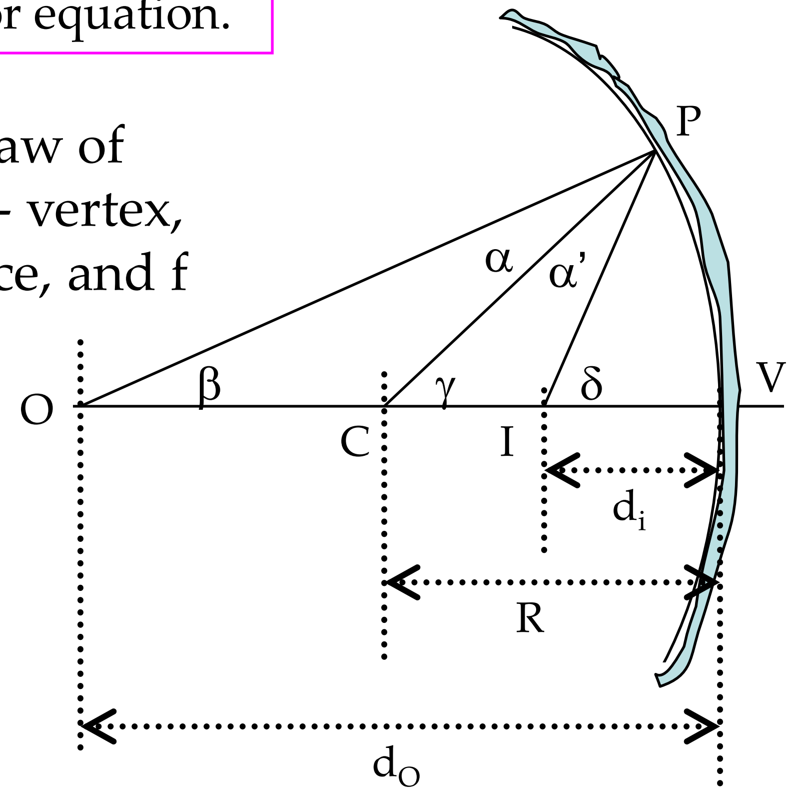
$$R - x = R(1 - \frac{y^2}{R^2})^{\frac{1}{2}}$$

$$R - x = R(1 - \frac{1}{2}\frac{y^2}{R^2})$$

$$x = \frac{y^2}{2R}$$

Now, let us get what is called the mirror equation.

Let O- object, I-image, α = α′ by the law of reflection, C - center of curvature, V - vertex, d_i - image distance, d_o - object distance, and f - focal length. So



$$\alpha + \beta = \gamma$$

$$\beta + 2\alpha = \delta, . \Rightarrow$$

$$\beta + \delta = 2\gamma$$

Assume small angle approximation

$$\beta + \delta = 2\gamma$$

$$\frac{PV}{d_0} + \frac{PV}{d_i} = 2\frac{PV}{R}$$

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{2}{R}$$

Note if $d_o = \infty$, then $d_i = f = R/2$.

The Spherical Mirror equation is

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$f = \frac{R}{2}$$

Let us look at the lens

Note O is the origin

$$P_{COF} = nd + f$$

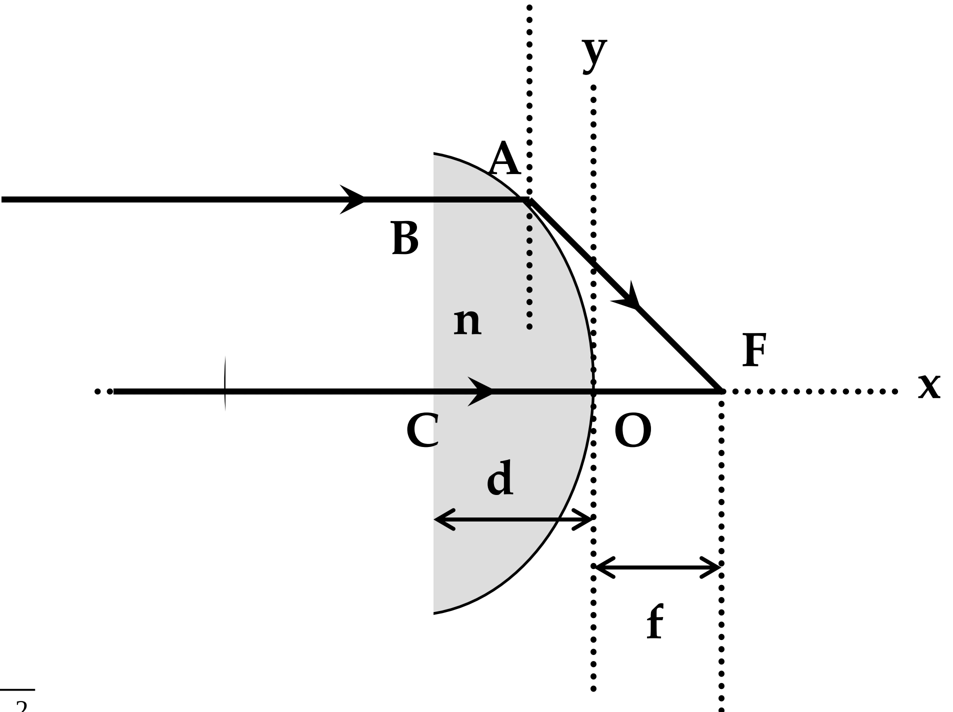
$$P_{BAF} = n(d+x) + \sqrt{y^2 + (f-x)^2}$$

Equate these two expressions

$$nd + f = n(d+x) + \sqrt{y^2 + (f-x)^2}$$

$$(f - nx)^2 = (f-x)^2 + y^2$$

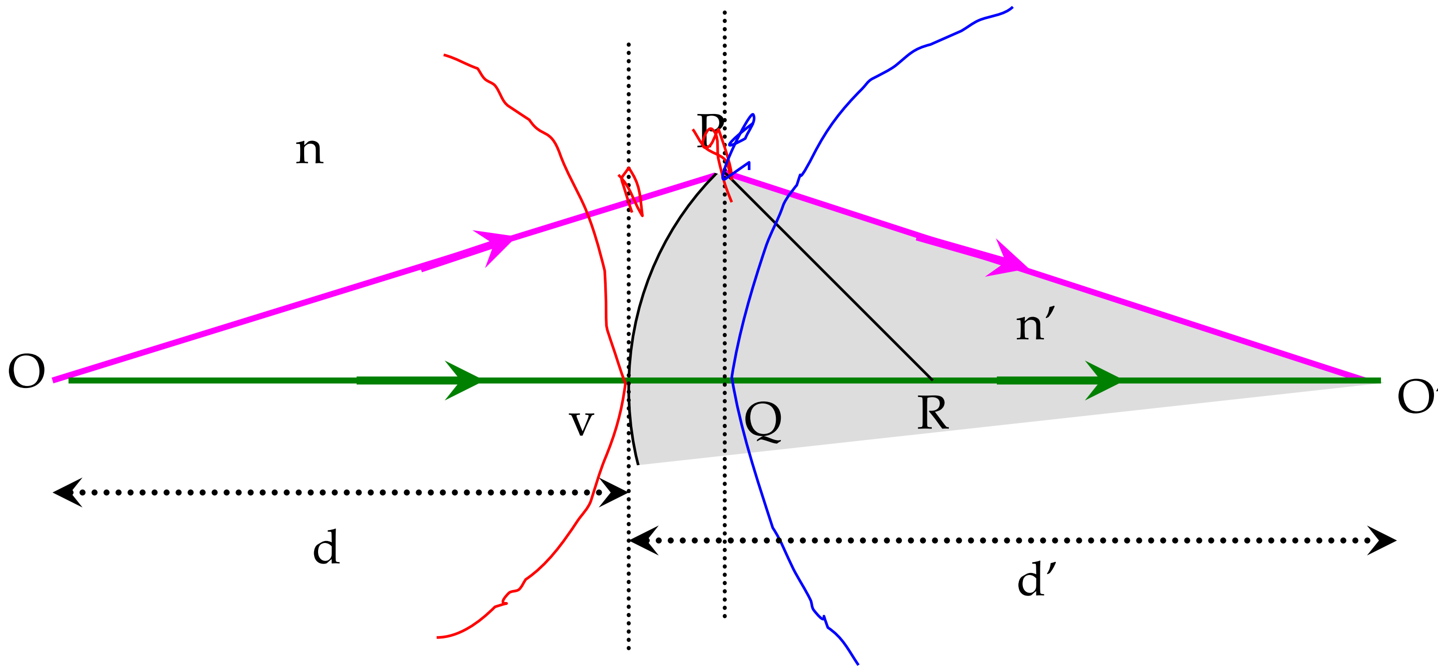
$$x = \frac{-f(1-n) - \sqrt{f^2(n-1)^2 + (n^2-1)y^2}}{n^2 - 1}$$



And this is an equation for hyperbola for **negative x**.
Try to plot it with Maple for f = 10 and n = 1.5

The point here again is ideal optical elements are not spherical. This is difficult to fabricate as I said before. They can be fabricated, but they will be expensive! Usually we work with small angles so that spherical surfaces will be good approximation to the ideal surfaces. Let us look at spherical surfaces again.

Imagine light goes from medium of index n to another medium of index n′ where the boundary is spherical. Again take two paths (purple and green) and equate their times.

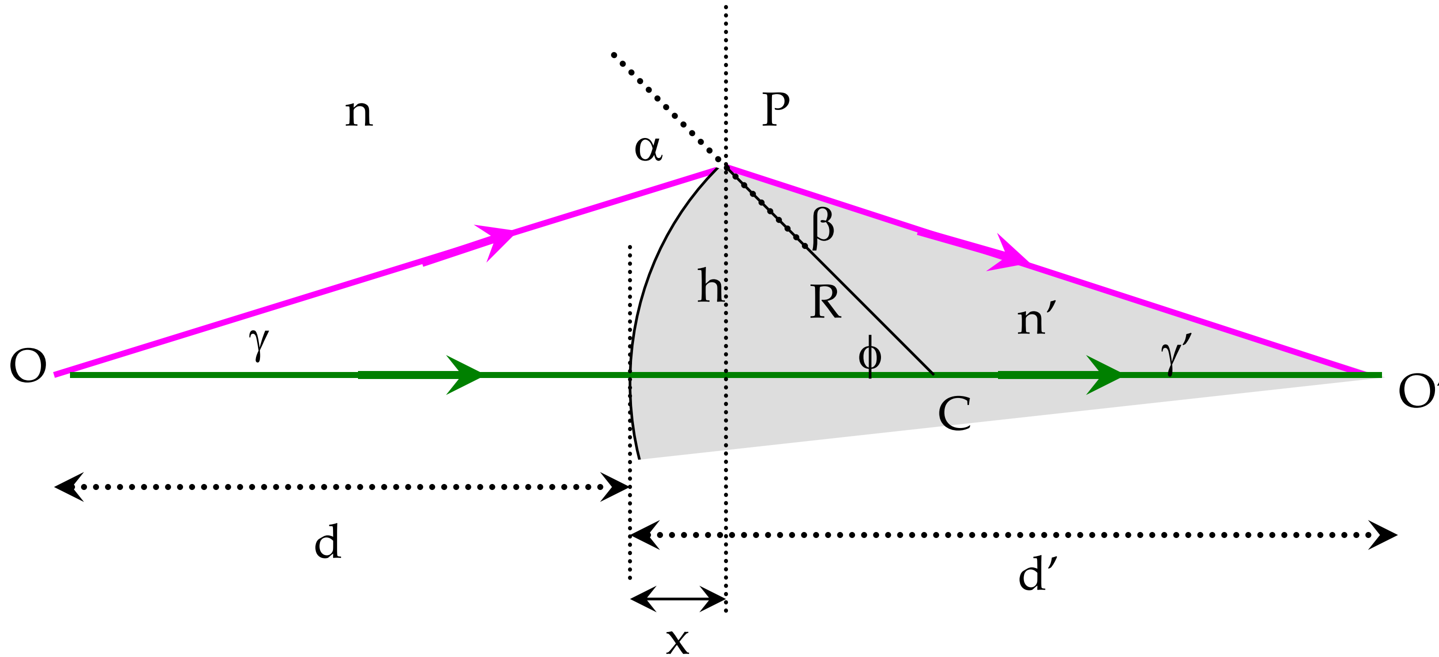


$$nd + n\frac{y^2}{2d} + n\frac{y^2}{2R} + n'(d' - \frac{y^2}{2R}) + n'\frac{y^2}{2d'} = nd + n'd'$$

Simplify
$$\frac{n'}{d'} + \frac{n}{d} = \frac{n' - n}{R}$$

The lens equation

The lens equation again



$$\alpha = \gamma + \phi$$
$$\phi = \beta + \gamma'$$
$$\beta = \phi - \gamma'$$

$$\tan(\gamma) = \frac{h}{d+x}$$
$$\tan(\gamma') = \frac{h}{d'-x}$$
$$\sin(\phi) = \frac{h}{R}$$

$$\sin(\alpha) \approx \alpha = \gamma + \phi \approx \frac{h}{d} + \frac{h}{R}$$
$$\sin(\beta) \approx \beta = \phi - \gamma' \approx \frac{h}{R} - \frac{h}{d'}$$

Use Snell's law

$$n.\sin(\alpha) = n'.\sin(\beta)$$

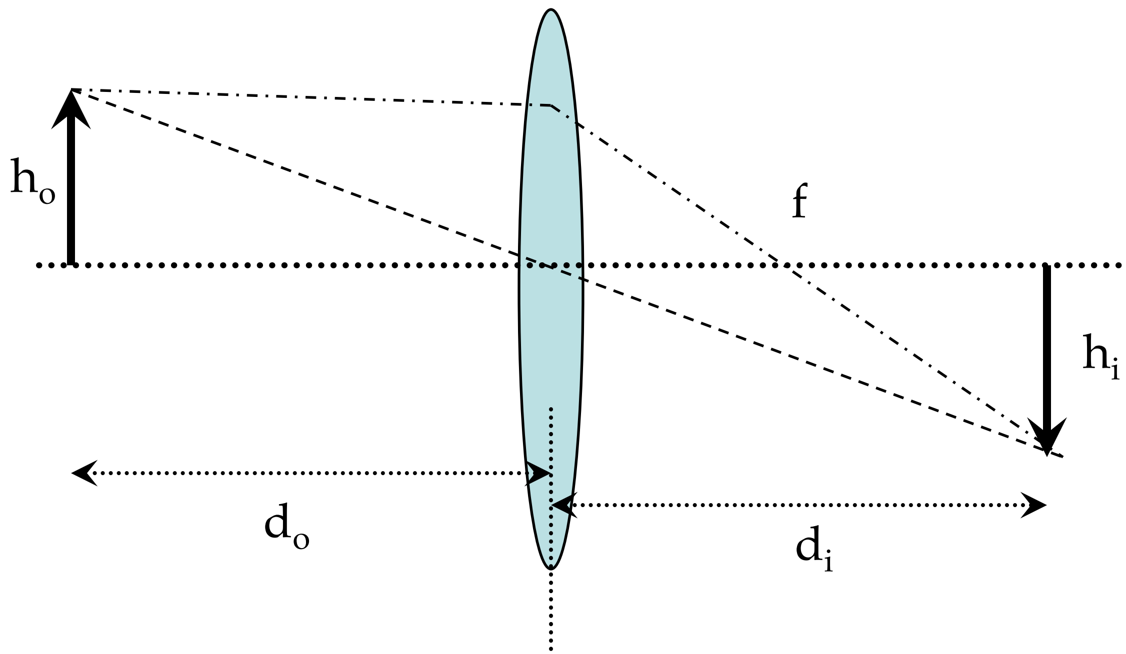
$$n.\sin(\alpha) = n'.\sin(\beta) \qquad n.(\frac{h}{d} + \frac{h}{R}) = n'(\frac{h}{R} - \frac{h}{d'}) \qquad \frac{n}{d} + \frac{n'}{d'} = \frac{n'-n}{R}$$

The thin lens equation

$$\frac{h_o}{d_o} = \frac{h_i}{d_i}$$

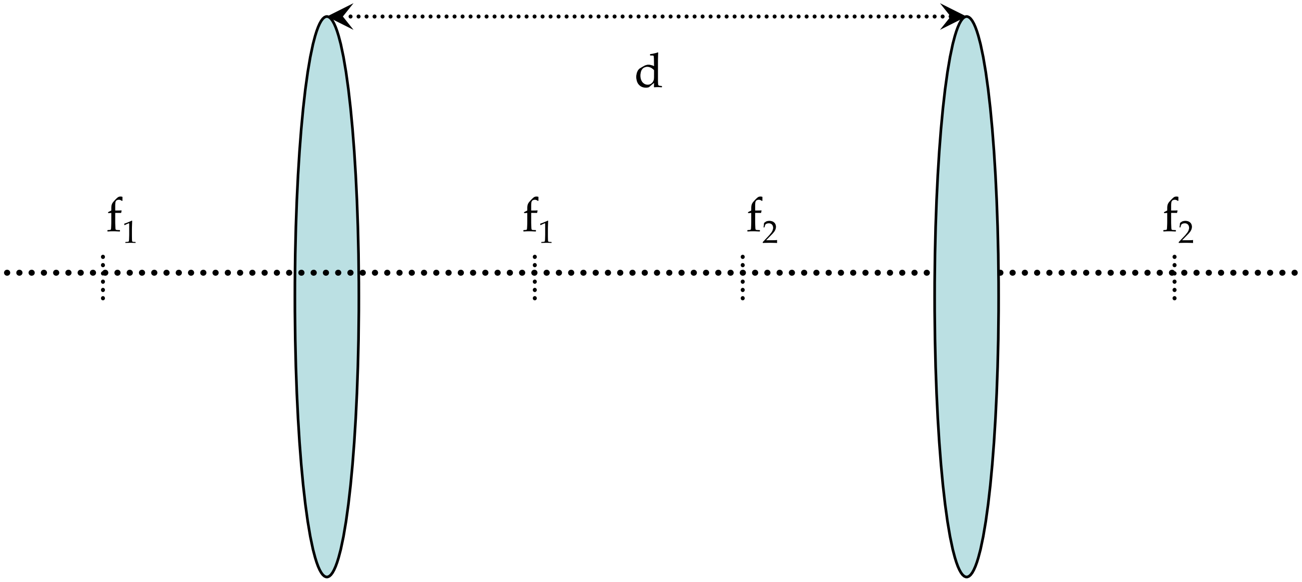
$$\frac{h_o}{f} = \frac{h_i}{d_i - f}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$



$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Lens combination



And as you know there is a tremendous applications of lenses, mirrors, and the all possible combinations. We will mention few and you should read about the rest as much as you can.

Why Fermat's Principle?

Why does light travel such that time is minimal? Actually the precise statement is that light travels such that time is extremum. This means $dt/dx = 0$, i.e. no first order contributions. Well, you might ask then how dos the light know which path is extrmum? It actually samples all of the paths and keep track of their lengths via their phases. This is exactly what is happening, as incredible as it may seem! The phase of a light ray, ϕ, is defined by $e^{i\phi} = e^{ikr}$. Different paths will have different r's which in turn will have different phases. Only the paths that are very close to each other will interfere constructively and the rest will cancel out. So the true path will be the one where there are no first order contributions to the phase when the path changes instantaneously, so there is nothing to cancel out – this is exactly the principle of extremal or least time.

Now, the geometrical approximation is where λ is very small and realizing that $k = 2\pi/\lambda$, so k is very large and small changes in r on average will cancel out because kr is large.

Okay, you say, but why should we add the phases, not multiply for example? That is because light is regarded as an electromagnetic wave. As a consequence, you learn the superposition principle which will dictate the addition of phases as you will see in the wave approach to light next.